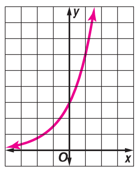
NAME ______________________ DATE __________ PERIOD _____

9-1 Skills Practice

Exponential Functions

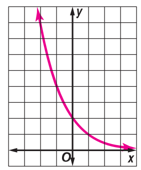
Sketch the graph of each function. Then state the function's domain and range.

1. $y = 3(2)^x$



domain: all real numbers; range: all positive numbers

2. $y = 2\left(\frac{1}{2}\right)^x$



domain: all real numbers; range: all positive numbers

Determine whether each function represents exponential *growth* or *decay*.

3. $y = 3(6)^x$ growth

4. $y = 2\left(\frac{9}{10}\right)^x$ decay

5. $y = 10^{-x}$ decay

6. $y = 2(2.5)^x$ growth

Write an exponential function whose graph passes through the given points.

7. (0, 1) and (−1, 3) $y = \left(\frac{1}{3}\right)^x$

8. (0, 4) and (1, 12) $y = 4(3)^x$

9. (0, 3) and (−1, 6) $y = 3\left(\frac{1}{2}\right)^x$

10. (0, 5) and (1, 15) $y = 5(3)^x$

11. (0, 0.1) and (1, 0.5) $y = 0.1(5)^x$

12. (0, 0.2) and (1, 1.6) $y = 0.2(8)^x$

Simplify each expression.

13. $(3^{\sqrt{3}})^{\sqrt{3}}$ 27

14. $(x^{\sqrt{2}})^{\sqrt{7}}$ $x^{\sqrt{14}}$

15. $5^{2\sqrt{3}} \cdot 5^{4\sqrt{3}}$ $5^{6\sqrt{3}}$

16. $x^{3\pi} \div x^{\pi}$ $x^{2\pi}$

Solve each equation or inequality. Check your solution.

17. $3^x > 9$ $x > 2$

18. $2^{2x+3} = 32$ 1

19. $49^x \le \frac{1}{7}$ $x \le -\frac{1}{2}$

20. $4^{3x-2} = 16$ $\frac{4}{3}$

21. $3^{2x+5} = 27^x$ 5

22. $27^x = 3^{2x+3}$ 3

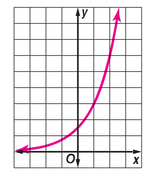
NAME ______________________ DATE __________ PERIOD _____

9-1 Practice

Exponential Functions

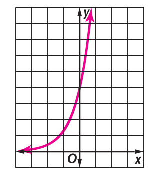
Sketch the graph of each function. Then state the function's domain and range.

1. $y = 1.5(2)^x$



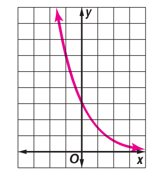
domain: all real numbers; range: all positive numbers

2. $y = 4(3)^x$



domain: all real numbers; range: all positive numbers

3. $y = 3(0.5)^x$



domain: all real numbers; range: all positive numbers

Determine whether each function represents exponential *growth* or *decay*.

4. $y = 5(0.6)^x$ decay

5. $y = 0.1(2)^x$ growth

6. $y = 5 \cdot 4^{-x}$ decay

Write an exponential function whose graph passes through the given points.

7. (0, 1) and (−1, 4) $y = \left(\frac{1}{4}\right)^x$

8. (0, 2) and (1, 10) $y = 2(5)^x$

9. (0, −3) and (1, −1.5) $y = -3(0.5)^x$

10. (0, 0.8) and (1, 1.6) $y = 0.8(2)^x$

11. (0, −0.4) and (2, −10) $y = -0.4(5)^x$

12. (0, π) and (3, 8π) $y = \pi(2)^x$

Simplify each expression.

13. $(2^{\sqrt{2}})^{\sqrt{8}}$ 16

14. $(n^{\sqrt{3}})^{\sqrt{75}}$ n^{15}

15. $y^{\sqrt{6}} \cdot y^{5\sqrt{6}}$ $y^{6\sqrt{6}}$

16. $13^{\sqrt{6}} \cdot 13^{\sqrt{24}}$ $13^{3\sqrt{6}}$

17. $n^3 \div n^{\pi}$ $n^{3-\pi}$

18. $125^{\sqrt{11}} \div 5^{\sqrt{11}}$ $5^{2\sqrt{11}}$

Solve each equation or inequality. Check your solution.

19. $3^{3x-5} > 81$ $x > 3$

20. $7^{6x} = 7^{2x-20}$ −5

21. $3^{6n-5} < 9^{4n-3}$ $n > \frac{1}{2}$

22. $9^{2x-1} = 27^{x+4}$ 14

23. $2^{3n-1} \ge \left(\frac{1}{8}\right)^n$ $n \ge \frac{1}{6}$

24. $16^{4n-1} = 128^{2n+1}$ $\frac{11}{2}$

BIOLOGY For Exercises 25 and 26, use the following information.

The initial number of bacteria in a culture is 12,000. The number after 3 days is 96,000.

25. Write an exponential function to model the population y of bacteria after x days. $y = 12{,}000(2)^x$

26. How many bacteria are there after 6 days? 768,000

27. EDUCATION A college with a graduating class of 4000 students in the year 2005 predicts that it will have a graduating class of 4862 in 4 years. Write an exponential function to model the number of students y in the graduating class t years after 2005. $y = 4000(1.05)^t$

NAME ______________________ DATE __________ PERIOD _____

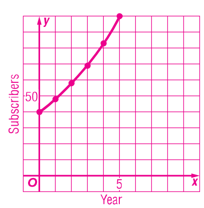
9-1 Word Problem Practice

Exponential Functions

1. GOLF BALLS A golf ball manufacturer packs 3 golf balls into a single package. Three of these packages make a gift box. Three gift boxes make a value pack. The display shelf is high enough to stack 3 value packs one on top of the other. Three such columns of value packs make up a display front. Three display fronts can be packed in a single shipping box and shipped to various retail stores. How many golf balls are in a single shipping box?
729

2. FOLDING Kay folds a piece of paper in half over and over until it is at least 25 layers thick. How many times does she fold the paper in half and how many layers are there?
5 times, producing 32 layers

3. SUBSCRIPTIONS Subscriptions to an online arts and crafts club have been increasing by 20% every year. The club began with 40 members. Make a graph of the number of subscribers over the first 5 years of the club's existence.



4. TENNIS SHOES The cost of a pair of tennis shoes increases about 5.1% every year. About how much would a $50 pair of tennis shoes cost 25 years from now?
$173.40

MONEY For Exercises 5–7, use the following information.

Sam opened a savings account that accrues compound interest at a rate of 3% annually. Let P be the initial amount Sam deposited and let t be the number of years the account has been open.

5. Write an equation to find A, the amount of money in the account after t years. Assume that Sam made more additional deposits and no withdrawals.
$\mathbf{A = P(1.03)^t}$

6. If Sam opened the account with $500 and made no deposits or withdrawals, how much is in the account 10 years later?
$671.96

7. What is the least number of years it would take for such an account to double in value?
24 years

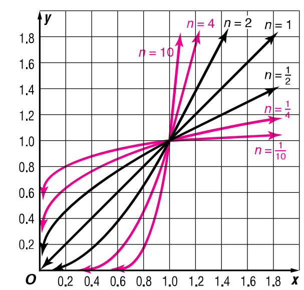
NAME ______________________ DATE __________ PERIOD _____

9-1 Enrichment

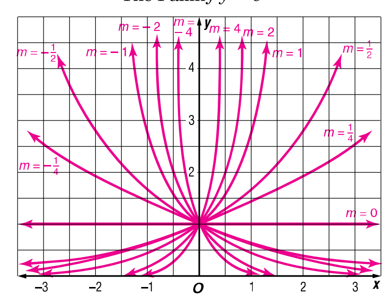
Families of Curves

Use these graphs for the problems below.

The Family $y = x^n$



The Family $y = e^{mx}$



1. Use the graph on the left to describe the relationship among the curves $y = x^{\frac{1}{2}}$, $y = x^1$, and $y = x^2$. **For $n = \frac{1}{2}$ and $n = 2$, the graphs are reflections of one another in the line with equation $y = x^1$.**

2. Graph $y = x^n$ for $n = \frac{1}{10}, \frac{1}{4}$, 4, and 10 on the grid with $y = x^{\frac{1}{2}}$, $y = x^1$, and $y = x^2$.
See students' graphs.

3. Which two regions in the first quadrant contain no points of the graphs of the family for $y = x^n$?
$\{(x, y) | x \geq 1 \text{ and } 0 < y \leq 1\}$ and $\{(x, y) | 0 < x \leq 1 \text{ and } y \geq 1\}$

4. On the right grid, graph the members of the family $y = e^{mx}$ for which $m = 1$ and $m = -1$.
See students' graphs.

5. Describe the relationship among these two curves and the y-axis.
the graphs for $m = 1$ and $m = -1$ are reflections in the y-axis.

6. Graph $y = e^{mx}$ for $m = 0, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 2$, and ± 4.
See students' graphs.

NAME ______________________ DATE __________ PERIOD _____

9-1 Graphing Calculator Activity

Regression Equation Lab

A graphing calculator can be used to determine a regression equation that best fits a set of data. This activity requires tiles labeled on one side, and a container.

Collect the Data

Step 1 Place the tiles on the desktop and count the total number. Record the total number. Then place the tiles in the container and gently shake.

Step 2 Pour the tiles onto the desktop, remove all the tiles with a label showing, and set these aside. Count the remaining tiles without the labels showing and return them to the container.

Step 3 Record the data in a table like this one.

Trials x	Number of tiles without label showing y
1	
2	

Step 4 Repeat step 2 and 3 until the number of tiles without labels is zero or the number remains constant.

Step 5 Take the tiles that were set aside in Step 2 and pour them out of the container onto the desktop. Remove the tiles without the label showing and count the tiles with the label showing. Repeat this process until all the tiles have been removed.

Step 6 Record the data in a table like this one.

Trials x	Number of tiles with the label showing y
1	
2	

Analyze the Data **1-6. Answers will vary.**

1. Enter trials in **L1** and number of tiles without label showing in **L2**. Enter trials in **L3** and number of tiles with the label showing in **L4**.

2. Use **[STATPLOT]** to make a scatter plot. Make a graph on paper for each plot. Record the window used. Describe the pattern of the points.

3. From the STAT **[CALC]** menu find the regression equation that best fits the data. Record the two closest equations, rounding values to the nearest hundredths. List and discuss the r and/or r^2 values. Also include the graphs in determining the best-fitting regression equation.

4. Sketch your best-fit regression equation choice for each scatter-plot on paper.

5. Describe any problems with the data or the regression equations.

6. Insert (0, total number of tiles) in the tables and the lists. Describe the effect on the graphs. What happens with **[PwrReg]** and **[ExpReg]** when this ordered pair is inserted? Explain why this occurs?

NAME ______________________ DATE __________ PERIOD _____

9-2 Reading to Learn Mathematics

Logarithms and Logarithmic Functions

Get Ready for the Lesson

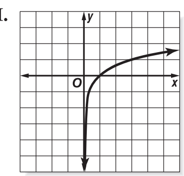
Read the introduction to Lesson 9-2 in your textbook.

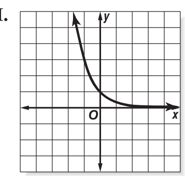
How many times louder than a whisper is normal conversation? **10^4 or 10,000 times**

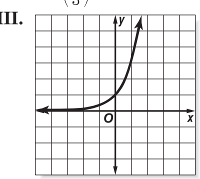
Read the Lesson

1. a. Write an exponential equation that is equivalent to $\log_3 81 = 4$. **$3^4 = 81$**
   b. Write a logarithmic equation that is equivalent to $25^{-\frac{1}{2}} = \frac{1}{5}$. **$\log_{25} \frac{1}{5} = -\frac{1}{2}$**
   c. Write an exponential equation that is equivalent to $\log_4 1 = 0$. **$4^0 = 1$**
   d. Write a logarithmic equation that is equivalent to $10^{-3} = 0.001$. **$\log_{10} 0.001 = -3$**
   e. What is the inverse of the function $y = 5^x$? **$y = \log_5 x$**
   f. What is the inverse of the function $y = \log_{10} x$? **$y = 10^x$**

2. Match each function with its graph.
   a. $y = 3^x$ **III** b. $y = \log_3 x$ **I** c. $y = \left(\frac{1}{3}\right)^x$ **II**

I.


II.


III.


3. Indicate whether each of the following statements about the exponential function $y = \log_5 x$ is *true* or *false*.
   a. The y-axis is an asymptote of the graph. **true**
   b. The domain is the set of all real numbers. **false**
   c. The graph contains the point (5, 0). **false**
   d. The range is the set of all real numbers. **true**
   e. The y-intercept is 1. **false**

Remember What You Learned

4. An important skill needed for working with logarithms is changing an equation between logarithmic and exponential forms. Using the words *base*, *exponent*, and *logarithm*, describe an easy way to remember and apply the part of the definition of logarithm that says, "$\log_b x = y$ if and only if $b^y = x$." **Sample answer: In these equations, *b* stands for *base*. In log form, *b* is the subscript, and in exponential form, *b* is the number that is raised to a power. A *logarithm* is an *exponent*, so *y*, which is the log in the first equation, becomes the exponent in the second equation.**

NAME ______________________ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention

Logarithms and Logarithmic Functions

Logarithmic Functions and Expressions

Definition of Logarithm with Base b	Let b and x be positive numbers, $b \neq 1$. The logarithm of x with base b is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.

The inverse of the exponential function $y = b^x$ is the **logarithmic function** $x = b^y$.
This function is usually written as $y = \log_b x$.

Properties of Logarithmic Functions	1. The function is continuous and one-to-one. 2. The domain is the set of all positive real numbers. 3. The y-axis is an asymptote of the graph. 4. The range is the set of all real numbers. 5. The graph contains the point (1, 0).

Example 1 **Write an exponential equation equivalent to $\log_3 243 = 5$.**

$3^5 = 243$

Example 2 **Write a logarithmic equation equivalent to $6^{-3} = \frac{1}{216}$.**

$\log_6 \frac{1}{216} = -3$

Example 3 **Evaluate $\log_8 16$.**

$8^{\frac{4}{3}} = 16$, so $\log_8 16 = \frac{4}{3}$.

Exercises

Write each equation in logarithmic form.

1. $2^7 = 128$ **$\log_2 128 = 7$**
2. $3^{-4} = \frac{1}{81}$ **$\log_3 \frac{1}{81} = -4$**
3. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$ **$\log_{\frac{1}{7}} \frac{1}{343} = 3$**

Write each equation in exponential form.

4. $\log_{15} 225 = 2$ **$15^2 = 225$**
5. $\log_3 \frac{1}{27} = -3$ **$3^{-3} = \frac{1}{27}$**
6. $\log_4 32 = \frac{5}{2}$ **$4^{\frac{5}{2}} = 32$**

Evaluate each expression.

7. $\log_4 64$ **3**
8. $\log_2 64$ **6**
9. $\log_{100} 100{,}000$ **2.5**
10. $\log_5 625$ **4**
11. $\log_{27} 81$ **$\frac{4}{3}$**
12. $\log_{25} 5$ **$\frac{1}{2}$**
13. $\log_2 \frac{1}{128}$ **−7**
14. $\log_{10} 0.00001$ **−5**
15. $\log_4 \frac{1}{32}$ **−2.5**

NAME ______________________ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention *(continued)*

Logarithms and Logarithmic Functions

Solve Logarithmic Equations and Inequalities

Logarithmic to Exponential Inequality	If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$. If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.
Property of Equality for Logarithmic Functions	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
Property of Inequality for Logarithmic Functions	If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example 1 **Solve $\log_2 2x = 3$.**

$\log_2 2x = 3$ Original equation
$2x = 2^3$ Definition of logarithm
$2x = 8$ Simplify.
$x = 4$ Simplify.
The solution is $x = 4$.

Example 2 **Solve $\log_5 (4x - 3) < 3$.**

$\log_5 (4x - 3) < 3$ Original equation
$0 < 4x - 3 < 5^3$ Logarithmic to exponential inequality
$3 < 4x < 125 + 3$ Addition Property of Inequalities
$\frac{3}{4} < x < 32$ Simplify.
The solution set is $\left\{x \middle| \frac{3}{4} < x < 32\right\}$.

Exercises

Solve each equation or inequality.

1. $\log_2 32 = 3x$ **$\frac{5}{3}$**
2. $\log_3 2c = -2$ **$\frac{1}{18}$**
3. $\log_{2x} 16 = -2$ **$\frac{1}{8}$**
4. $\log_{25}\left(\frac{x}{2}\right) = \frac{1}{2}$ **10**
5. $\log_4 (5x + 1) = 2$ **3**
6. $\log_8 (x - 5) = \frac{2}{3}$ **9**
7. $\log_4 (3x - 1) = \log_4 (2x + 3)$ **4**
8. $\log_2 (x^2 - 6) = \log_2 (2x + 2)$ **4**
9. $\log_{x+4} 27 = 3$ **−1**
10. $\log_2 (x + 3) = 4$ **13**
11. $\log_x 1000 = 3$ **10**
12. $\log_8 (4x + 4) = 2$ **15**
13. $\log_2 2x > 2$ **$x > 2$**
14. $\log_5 x > 2$ **$x > 25$**
15. $\log_2 (3x + 1) < 4$ **$-\frac{1}{3} < x < 5$**
16. $\log_4 (2x) > -\frac{1}{2}$ **$x > \frac{1}{4}$**
17. $\log_3 (x + 3) < 3$ **$-3 < x < 24$**
18. $\log_{27} 6x > \frac{2}{3}$ **$x > \frac{3}{2}$**

NAME ______________________ DATE ____________ PERIOD _____

9-2 Skills Practice

Logarithms and Logarithmic Functions

Write each equation in logarithmic form.

1. $2^3 = 8$ **$\log_2 8 = 3$**

2. $3^2 = 9$ **$\log_3 9 = 2$**

3. $8^{-2} = \frac{1}{64}$ **$\log_8 \frac{1}{64} = -2$**

4. $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ **$\log_{\frac{1}{3}} \frac{1}{9} = 2$**

Write each equation in exponential form.

5. $\log_3 243 = 5$ **$3^5 = 243$**

6. $\log_4 64 = 3$ **$4^3 = 64$**

7. $\log_9 3 = \frac{1}{2}$ **$9^{\frac{1}{2}} = 3$**

8. $\log_5 \frac{1}{25} = -2$ **$5^{-2} = \frac{1}{25}$**

Evaluate each expression.

9. $\log_5 25$ **2**

10. $\log_9 3$ **$\frac{1}{2}$**

11. $\log_{10} 1000$ **3**

12. $\log_{125} 5$ **$\frac{1}{3}$**

13. $\log_4 \frac{1}{64}$ **−3**

14. $\log_5 \frac{1}{625}$ **−4**

15. $\log_8 8^3$ **3**

16. $\log_{27} \frac{1}{3}$ **$-\frac{1}{3}$**

Solve each equation or inequality. Check your solutions.

17. $\log_3 x = 5$ **243**

18. $\log_2 x = 3$ **8**

19. $\log_4 y < 0$ **$0 < y < 1$**

20. $\log_{\frac{1}{4}} x = 3$ **$\frac{1}{64}$**

21. $\log_2 n > -2$ **$n > \frac{1}{4}$**

22. $\log_b 3 = \frac{1}{2}$ **9**

23. $\log_6 (4x + 12) = 2$ **6**

24. $\log_2 (4x - 4) > 5$ **$x > 9$**

25. $\log_3 (x + 2) = \log_3 (3x)$ **1**

26. $\log_6 (3y - 5) \geq \log_6 (2y + 3)$ **$y \geq 8$**

NAME ______________________ DATE ____________ PERIOD _____

9-2 Practice

Logarithms and Logarithmic Functions

Write each equation in logarithmic form.

1. $5^3 = 125$ **$\log_5 125 = 3$**

2. $7^0 = 1$ **$\log_7 1 = 0$**

3. $3^4 = 81$ **$\log_3 81 = 4$**

4. $3^{-4} = \frac{1}{81}$ **$\log_3 \frac{1}{81} = -4$**

5. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ **$\log_{\frac{1}{4}} \frac{1}{64} = 3$**

6. $7776^{\frac{1}{5}} = 6$ **$\log_{7776} 6 = \frac{1}{5}$**

Write each equation in exponential form.

7. $\log_6 216 = 3$ **$6^3 = 216$**

8. $\log_2 64 = 6$ **$2^6 = 64$**

9. $\log_3 \frac{1}{81} = -4$ **$3^{-4} = \frac{1}{81}$**

10. $\log_{10} 0.00001 = -5$ **$10^{-5} = 0.00001$**

11. $\log_{25} 5 = \frac{1}{2}$ **$25^{\frac{1}{2}} = 5$**

12. $\log_{32} 8 = \frac{3}{5}$ **$32^{\frac{3}{5}} = 8$**

Evaluate each expression.

13. $\log_3 81$ **4**

14. $\log_{10} 0.0001$ **−4**

15. $\log_2 \frac{1}{16}$ **−4**

16. $\log_{\frac{1}{3}} 27$ **−3**

17. $\log_9 1$ **0**

18. $\log_8 4$ **$\frac{2}{3}$**

19. $\log_7 \frac{1}{49}$ **−2**

20. $\log_6 6^4$ **4**

21. $\log_3 \frac{1}{3}$ **−1**

22. $\log_4 \frac{1}{256}$ **−4**

23. $\log_9 9^{(n+1)}$ **$n + 1$**

24. $2^{\log_2 32}$ **32**

Solve each equation or inequality. Check your solutions.

25. $\log_{10} n = -3$ **$\frac{1}{1000}$**

26. $\log_4 x > 3$ **$x > 64$**

27. $\log_4 x = \frac{3}{2}$ **8**

28. $\log_{\frac{1}{5}} x = -3$ **125**

29. $\log_7 q < 0$ **$0 < q < 1$**

30. $\log_6 (2y + 8) \geq 2$ **$y \geq 14$**

31. $\log_y 16 = -4$ **$\frac{1}{2}$**

32. $\log_n \frac{1}{8} = -3$ **2**

33. $\log_b 1024 = 5$ **4**

34. $\log_8 (3x + 7) < \log_8 (7x + 4)$ **$x > \frac{3}{4}$**

35. $\log_7 (8x + 20) = \log_7 (x + 6)$ **−2**

36. $\log_3 (x^2 - 2) = \log_3 x$ **2**

37. SOUND An equation for loudness, in decibels, is $L = 10 \log_{10} R$, where R is the relative intensity of the sound. Sounds that reach levels of 120 decibels or more are painful to humans. What is the relative intensity of 120 decibels? **10^{12}**

38. INVESTING Maria invests $1000 in a savings account that pays 4% interest compounded annually. The value of the account A at the end of five years can be determined from the equation $\log A = \log[1000(1 + 0.04)^5]$. Find the value of A to the nearest dollar. **$1217**

NAME ______________________ DATE ___________ PERIOD _____

9-2 Word Problem Practice

Logarithms and Logarithmic Functions

1. **FISH** The population of silver carp has been growing in the Mississippi River. About every 3 years, the population doubles. Write logarithmic expression that gives the number of years it will take for the population to increase by a factor of ten.
$3\log_2 10$

2. **POWERS** Haley tries to solve the equation $\log_4 2x = 32$. She got the wrong answer. What was her mistake? What should the correct answer be?

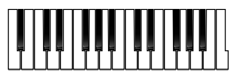
1.	$\log_4 2x = 5$
2.	$2x = 4^5$
3.	$x = 2^5$
4.	$x = 32$

From step 2 to step 3, Haley divided the equation by 2 incorrectly. The correct answer is 512.

3. **DIGITS** A computer programmer wants to write a formula that tells how many digits there are in a number n, where n is a positive integer. For example, if $n = 343$, the formula should evaluate to 3 and if $n = 10{,}000$, the formula should evaluate to 5. Suppose $8 \leq \log_{10} n < 9$. How many digits does n have?
9

4. **LOGARITHMS** Pauline knows that $\log_b x = 3$ and $\log_b y = 5$. She knows that this is the same as knowing that $b^3 = x$ and $b^5 = y$. Multiply these two equations together and rewrite it as an equation involving logarithms. What is $\log_b xy$?
$b^3b^5 = xy$, or $b^8 = xy$; in other words, $\log_b xy = 8$

MUSIC For Exercises 5 and 6, use the following information.



The first note on a piano keyboard corresponds to a pitch with a frequency of 27.5 cycles per second. With every successive note you go up the white and black keys of a piano, the pitch multiplies by a factor of $\sqrt[12]{2}$. The formula for the frequency of the pitch sounded when the nth note up the keyboard is played is given by

$$n = 1 + 12\log_2 \frac{f}{27.5}.$$

5. The pitch that orchestras tune to is the A above middle C. It has a frequency of 440 cycles per second. How many notes up the piano keyboard is this A?
49

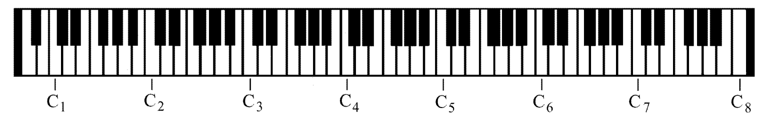
6. Another pitch on the keyboard has a frequency of 1760 cycles per second. How many notes up the keyboard will this be found?
73

NAME ______________________ DATE ___________ PERIOD _____

9-2 Enrichment

Musical Relationships

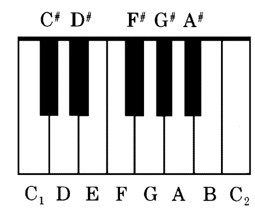
The frequencies of notes that are one octave apart in a musical scale are related by an exponential equation. For the eight C notes on a piano, the equation is $C_n = C_1 2^{n-1}$, where C_n represents the frequency of note C_n.



1. Find the relationship between C_1 and C_2. **$C_2 = 2C_1$**

2. Find the relationship between C_1 and C_4. **$C_4 = 8C_1$**

The frequencies of consecutive notes are related by a common ratio r. The general equation is $f_n = f_1 r^{n-1}$.

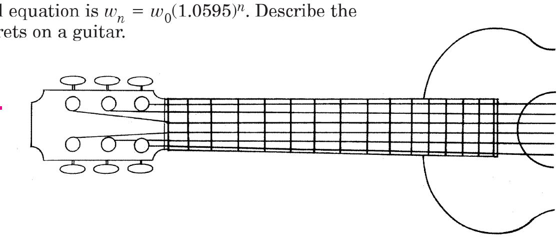


3. If the frequency of middle C is 261.6 cycles per second and the frequency of the next higher C is 523.2 cycles per second, find the common ratio r. (*Hint:* The two C's are 12 notes apart.) Write the answer as a radical expression.
$r = \sqrt[12]{2}$

4. Substitute decimal values for r and f_1 to find a specific equation for f_n.
$f_n = 261.1(1.05946)^{n-1}$

5. Find the frequency of F# above middle C.
$f_7 = 261.6(1.05946)^6 \approx 369.95$

6. Frets are a series of ridges placed across the fingerboard of a guitar. They are spaced so that the sound made by pressing a string against one fret has about 1.0595 times the wavelength of the sound made by using the next fret. The general equation is $w_n = w_0(1.0595)^n$. Describe the arrangement of the frets on a guitar.
The frets are spaced in a logarithmic scale.



NAME ______________ DATE ________ PERIOD _____

9-3 Lesson Reading Guide

Properties of Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-3 in your textbook.

1. Find the value of each of the following.
   a. $\log_5 125$ **3** **b.** $\log_5 5$ **1** **c.** $\log_5 (125 \div 5)$ **2**
2. Which of the following statements is true? **B**
   A. $\log_5 (125 \div 5) = (\log_5 125) \div (\log_5 5)$
   B. $\log_5 (125 \div 5) = \log_5 125 - \log_5 5$

Read the Lesson

1. Each of the properties of logarithms can be stated in words or in symbols. Complete the statements of these properties in words.
   a. The logarithm of a quotient is the **difference** of the logarithms of the **numerator** and the **denominator**.
   b. The logarithm of a power is the **product** of the logarithm of the base and the **exponent**.
   c. The logarithm of a product is the **sum** of the logarithms of its **factors**.
2. State whether each of the following equations is *true* or *false*. If the statement is true, name the property of logarithms that is illustrated.
   a. $\log_3 10 = \log_3 30 - \log_3 3$ **true; Quotient Property**
   b. $\log_4 12 = \log_4 4 + \log_4 8$ **false**
   c. $\log_2 81 = 2 \log_2 9$ **true; Power Property**
   d. $\log_8 30 = \log_8 5 \cdot \log_8 6$ **false**
3. The algebraic process of solving the equation $\log_2 x + \log_2 (x + 2) = 3$ leads to "$x = -4$ or $x = 2$." Does this mean that both −4 and 2 are solutions of the logarithmic equation? Explain your reasoning. **Sample answer: No; 2 is a solution because it checks: $\log_2 2 + \log_2 (2 + 2) = \log_2 2 + \log_2 4 = 1 + 2 = 3$. However, because $\log_2 (-4)$ and $\log_2 (-2)$ are undefined, −4 is an extraneous solution and must be eliminated. The only solution is 2.**

Remember What You Learned

4. A good way to remember something is to relate it something you already know. Use words to explain how the Product Property for exponents can help you remember the product property for logarithms. **Sample answer: When you *multiply* two numbers or expressions with the same base, you *add* the exponents and keep the *same base*. Logarithms are exponents, so to find the logarithm of a *product*, you *add* the logarithms of the factors, keeping the *same base*.**

NAME ______________ DATE ________ PERIOD _____

9-3 Study Guide and Intervention

Properties of Logarithms

Properties of Logarithms Properties of exponents can be used to develop the following properties of logarithms.

Product Property of Logarithms	For all positive numbers *m*, *n*, and *b*, where $b \neq 1$, $\log_b mn = \log_b m + \log_b n$.
Quotient Property of Logarithms	For all positive numbers *m*, *n*, and *b*, where $b \neq 1$, $\log_b \frac{m}{n} = \log_b m - \log_b n$.
Power Property of Logarithms	For any real number *p* and positive numbers *m* and *b*, where $b \neq 1$, $\log_b m^p = p \log_b m$.

Example **Use $\log_3 28 \approx 3.0331$ and $\log_3 4 \approx 1.2619$ to approximate the value of each expression.**

a. $\log_3 36$

$$\begin{aligned}\log_3 36 &= \log_3 (3^2 \cdot 4)\\ &= \log_3 3^2 + \log_3 4\\ &= 2 + \log_3 4\\ &\approx 2 + 1.2619\\ &\approx 3.2619\end{aligned}$$

b. $\log_3 7$

$$\begin{aligned}\log_3 7 &= \log_3 \left(\frac{28}{4}\right)\\ &= \log_3 28 - \log_3 4\\ &\approx 3.0331 - 1.2619\\ &\approx 1.7712\end{aligned}$$

c. $\log_3 256$

$$\begin{aligned}\log_3 256 &= \log_3 (4^4)\\ &= 4 \cdot \log_3 4\\ &\approx 4(1.2619)\\ &\approx 5.0476\end{aligned}$$

Exercises

Use $\log_{12} 3 \approx 0.4421$ and $\log_{12} 7 \approx 0.7831$ to evaluate each expression.

1. $\log_{12} 21$ **1.2252**
2. $\log_{12} \frac{7}{3}$ **0.3410**
3. $\log_{12} 49$ **1.5662**
4. $\log_{12} 36$ **1.4421**
5. $\log_{12} 63$ **1.6673**
6. $\log_{12} \frac{27}{49}$ **−0.2399**
7. $\log_{12} \frac{81}{49}$ **0.2022**
8. $\log_{12} 16{,}807$ **3.9155**
9. $\log_{12} 441$ **2.4504**

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to evaluate each expression.

10. $\log_5 12$ **1.5440**
11. $\log_5 100$ **2.8614**
12. $\log_5 0.75$ **−0.1788**
13. $\log_5 144$ **3.0880**
14. $\log_5 \frac{27}{16}$ **0.3250**
15. $\log_5 375$ **3.6826**
16. $\log_5 1.\overline{3}$ **0.1788**
17. $\log_5 \frac{9}{16}$ **−0.3576**
18. $\log_5 \frac{81}{5}$ **1.7304**

NAME ______________________ DATE ____________ PERIOD _____

9-3 Study Guide and Intervention *(continued)*

Properties of Logarithms

Solve Logarithmic Equations You can use the properties of logarithms to solve equations involving logarithms.

Example **Solve each equation.**

a. $2 \log_3 x - \log_3 4 = \log_3 25$

$2 \log_3 x - \log_3 4 = \log_3 25$	Original equation
$\log_3 x^2 - \log_3 4 = \log_3 25$	Power Property
$\log_3 \frac{x^2}{4} = \log_3 25$	Quotient Property
$\frac{x^2}{4} = 25$	Property of Equality for Logarithmic Functions
$x^2 = 100$	Multiply each side by 4.
$x = \pm 10$	Take the square root of each side.

Since logarithms are undefined for $x < 0$, -10 is an extraneous solution. The only solution is 10.

b. $\log_2 x + \log_2 (x + 2) = 3$

$\log_2 x + \log_2 (x + 2) = 3$	Original equation
$\log_2 x(x + 2) = 3$	Product Property
$x(x + 2) = 2^3$	Definition of logarithm
$x^2 + 2x = 8$	Distributive Property
$x^2 + 2x - 8 = 0$	Subtract 8 from each side.
$(x + 4)(x - 2) = 0$	Factor.
$x = 2$ or $x = -4$	Zero Product Property

Since logarithms are undefined for $x < 0$, -4 is an extraneous solution. The only solution is 2.

Exercises

Solve each equation. Check your solutions.

1. $\log_5 4 + \log_5 2x = \log_5 24$ **3**

2. $3 \log_4 6 - \log_4 8 = \log_4 x$ **27**

3. $\frac{1}{2} \log_6 25 + \log_6 x = \log_6 20$ **4**

4. $\log_2 4 - \log_2 (x + 3) = \log_2 8$ $-\frac{5}{2}$

5. $\log_6 2x - \log_6 3 = \log_6 (x - 1)$ **3**

6. $2 \log_4 (x + 1) = \log_4 (11 - x)$ **2**

7. $\log_2 x - 3 \log_2 5 = 2 \log_2 10$ **12,500**

8. $3 \log_2 x - 2 \log_2 5x = 2$ **100**

9. $\log_3 (c + 3) - \log_3 (4c - 1) = \log_3 5$ $\frac{8}{19}$

10. $\log_5 (x + 3) - \log_5 (2x - 1) = 2$ $\frac{4}{7}$

NAME ______________________ DATE ____________ PERIOD _____

9-3 Skills Practice

Properties of Logarithms

Use $\log_2 3 \approx 1.5850$ and $\log_2 5 \approx 2.3219$ to approximate the value of each expression.

1. $\log_2 25$ **4.6438**

2. $\log_2 27$ **4.755**

3. $\log_2 \frac{3}{5}$ **−0.7369**

4. $\log_2 \frac{5}{3}$ **0.7369**

5. $\log_2 15$ **3.9069**

6. $\log_2 45$ **5.4919**

7. $\log_2 75$ **6.2288**

8. $\log_2 0.6$ **−0.7369**

9. $\log_2 \frac{1}{3}$ **−1.5850**

10. $\log_2 \frac{9}{5}$ **0.8481**

Solve each equation. Check your solutions.

11. $\log_{10} 27 = 3 \log_{10} x$ **3**

12. $3 \log_7 4 = 2 \log_7 b$ **8**

13. $\log_4 5 + \log_4 x = \log_4 60$ **12**

14. $\log_6 2c + \log_6 8 = \log_6 80$ **5**

15. $\log_5 y - \log_5 8 = \log_5 1$ **8**

16. $\log_2 q - \log_2 3 = \log_2 7$ **21**

17. $\log_9 4 + 2 \log_9 5 = \log_9 w$ **100**

18. $3 \log_8 2 - \log_8 4 = \log_8 b$ **2**

19. $\log_{10} x + \log_{10} (3x - 5) = \log_{10} 2$ **2**

20. $\log_4 x + \log_4 (2x - 3) = \log_4 2$ **2**

21. $\log_3 d + \log_3 3 = 3$ **9**

22. $\log_{10} y - \log_{10} (2 - y) = 0$ **1**

23. $\log_2 s + 2 \log_2 5 = 0$ $\frac{1}{25}$

24. $\log_2 (x + 4) - \log_2 (x - 3) = 3$ **4**

25. $\log_4 (n + 1) - \log_4 (n - 2) = 1$ **3**

26. $\log_5 10 + \log_5 12 = 3 \log_5 2 + \log_5 a$ **15**

NAME ______________________ DATE __________ PERIOD _____

9-3 Practice
Properties of Logarithms

Use $\log_{10} 5 \approx 0.6990$ and $\log_{10} 7 \approx 0.8451$ to approximate the value of each expression.

1. $\log_{10} 35$ **1.5441**
2. $\log_{10} 25$ **1.3980**
3. $\log_{10} \frac{7}{5}$ **0.1461**
4. $\log_{10} \frac{5}{7}$ **−0.1461**
5. $\log_{10} 245$ **2.3892**
6. $\log_{10} 175$ **2.2431**
7. $\log_{10} 0.2$ **−0.6990**
8. $\log_{10} \frac{25}{7}$ **0.5529**

Solve each equation. Check your solutions.

9. $\log_7 n = \frac{2}{3} \log_7 8$ **4**
10. $\log_{10} u = \frac{3}{2} \log_{10} 4$ **8**
11. $\log_6 x + \log_6 9 = \log_6 54$ **6**
12. $\log_8 48 - \log_8 w = \log_8 4$ **12**
13. $\log_9 (3u + 14) - \log_9 5 = \log_9 2u$ **2**
14. $4 \log_2 x + \log_2 5 = \log_2 405$ **3**
15. $\log_3 y = -\log_3 16 + \frac{1}{3} \log_3 64$ **$\frac{1}{4}$**
16. $\log_2 d = 5 \log_2 2 - \log_2 8$ **4**
17. $\log_{10} (3m - 5) + \log_{10} m = \log_{10} 2$ **2**
18. $\log_{10} (b + 3) + \log_{10} b = \log_{10} 4$ **1**
19. $\log_8 (t + 10) - \log_8 (t - 1) = \log_8 12$ **2**
20. $\log_3 (a + 3) + \log_3 (a + 2) = \log_3 6$ **0**
21. $\log_{10} (r + 4) - \log_{10} r = \log_{10} (r + 1)$ **2**
22. $\log_4 (x^2 - 4) - \log_4 (x + 2) = \log_4 1$ **3**
23. $\log_{10} 4 + \log_{10} w = 2$ **25**
24. $\log_8 (n - 3) + \log_8 (n + 4) = 1$ **4**
25. $3 \log_5 (x^2 + 9) - 6 = 0$ **±4**
26. $\log_{16} (9x + 5) - \log_{16} (x^2 - 1) = \frac{1}{2}$ **3**
27. $\log_6 (2x - 5) + 1 = \log_6 (7x + 10)$ **8**
28. $\log_2 (5y + 2) - 1 = \log_2 (1 - 2y)$ **0**
29. $\log_{10} (c^2 - 1) - 2 = \log_{10} (c + 1)$ **101**
30. $\log_7 x + 2 \log_7 x - \log_7 3 = \log_7 72$ **6**

31. **SOUND** Recall that the loudness L of a sound in decibels is given by $L = 10 \log_{10} R$, where R is the sound's relative intensity. If the intensity of a certain sound is tripled, by how many decibels does the sound increase? **about 4.8 db**

32. **EARTHQUAKES** An earthquake rated at 3.5 on the Richter scale is felt by many people, and an earthquake rated at 4.5 may cause local damage. The Richter scale magnitude reading m is given by $m = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion. How many times greater is the amplitude of an earthquake that measures 4.5 on the Richter scale than one that measures 3.5? **10 times**

NAME ______________________ DATE __________ PERIOD _____

9-3 Word Problem Practice
Properties of Logarithms

1. **MENTAL COMPUTATION** Jessica has memorized $\log_5 2 \approx 0.4307$ and $\log_5 3 \approx 0.6826$. Using this information, to the nearest thousandth, what power of 5 is equal to 6?
1.113

2. **POWERS** A chemist is formulating an acid. The pH of a solution is given by

$$-\log_{10} C,$$

where C is the concentration of hydrogen ions. If the concentration of hydrogen ions is increased by a factor of 100, what happens to the pH of the solution?
The pH decreases by 2.

3. **LUCKY MATH** Frank is solving a problem involving logarithms. He does everything correctly except for one thing. He mistakenly writes

$$\log_2 a + \log_2 b = \log_2 (a + b).$$

However, after substituting the values for a and b in his problem, he amazingly still gets the right answer! The value of a was 11. What must the value of b have been?
1.1

4. **LENGTHS** Charles has two poles. One pole has length equal to $\log_7 21$ and the other has length equal to $\log_7 25$. Express the length of both poles joined end to end as the logarithm of a single number.
$\log_7 525$

SIZE For Exercises 5-7, use the following information.

Alicia wanted to try to quantify the terms *puny, tiny, small, medium, large, big, huge,* and *humongous*. She picked a number of objects and classified them with these adjectives of size. She noticed that the scale seemed exponential. Therefore, she came up with the following definition. Define S to be $\frac{1}{3} \log_3 V$, where V is volume in cubic feet.

Then use the following table to find the appropriate adjective.

S satisfies	Adjective
$-2 \le S < -1$	tiny
$-1 \le S < 0$	small
$0 \le S < 1$	medium
$1 \le S < 2$	large
$2 \le S < 3$	big
$3 \le S < 4$	huge

5. Derive an expression for S applied to a cube in terms of ℓ where ℓ is the side length of a cube.
$\log_3 \ell$

6. How many cubes, each one foot on a side, would have to be put together to get an object that Alicia would call "big"?
729

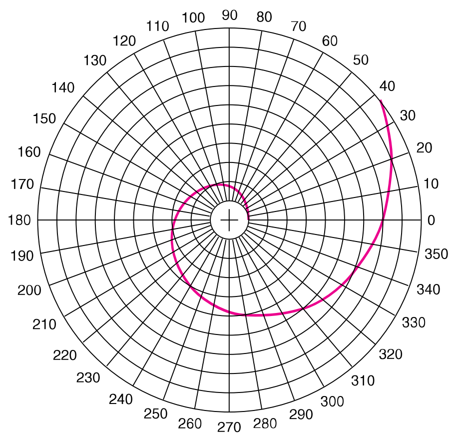
7. How likely is it that a large object attached to a big object would result in a huge object, according to Alicia's scale.
Not very likely; most likely the result will be big, not huge.

NAME ______________________ DATE ____________ PERIOD _____

9-3 Enrichment

Spirals

Consider an angle in standard position with its vertex at a point O called the pole. Its initial side is on a coordinatized axis called the *polar axis*. A point P on the terminal side of the angle is named by the *polar coordinates* (r, θ), where r is the directed distance of the point from O and θ is the measure of the angle. Graphs in this system may be drawn on polar coordinate paper such as the kind shown below.



1. Use a calculator to complete the table for $\log_2 r = \frac{\theta}{120}$.

(*Hint:* To find θ on a calculator, press 120 [×] [LOG] r [)] [÷] [LOG] 2 [)].)

r	1	2	3	4	5	6	7	8
θ	0°	120°	190°	240°	279°	310°	337°	360°

2. Plot the points found in Exercise 1 on the grid above and connect to form a smooth curve.

This type of spiral is called a logarithmic spiral because the angle measures are proportional to the logarithms of the radii.

NAME ______________________ DATE ____________ PERIOD _____

9-4 Lesson Reading Guide

Common Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-4 in your textbook.

Which substance is more acidic, milk or tomatoes? **tomatoes**

Read the Lesson

1. Rhonda used the following keystrokes to enter an expression on her graphing calculator:

[LOG] 17 [)] [ENTER]

The calculator returned the result 1.230448921.
Which of the following conclusions are correct? **a, c, and d**

a. The base 10 logarithm of 17 is about 1.2304.

b. The base 17 logarithm of 10 is about 1.2304.

c. The common logarithm of 17 is about 1.230449.

d. $10^{1.230448921}$ is very close to 17.

e. The common logarithm of 17 is exactly 1.230448921.

2. Match each expression from the first column with an expression from the second column that has the same value.

a. $\log_2 2$ **iv**	**i.** $\log_4 1$
b. $\log 12$ **iii**	**ii.** $\log_2 8$
c. $\log_3 1$ **i**	**iii.** $\log_{10} 12$
d. $\log_5 \frac{1}{5}$ **v**	**iv.** $\log_5 5$
e. $\log 1000$ **ii**	**v.** $\log 0.1$

3. Calculators do not have keys for finding base 8 logarithms directly. However, you can use a calculator to find $\log_8 20$ if you apply the **change of base** formula.
Which of the following expressions are equal to $\log_8 20$? **B and C**

A. $\log_{20} 8$ **B.** $\frac{\log_{10} 20}{\log_{10} 8}$ **C.** $\frac{\log 20}{\log 8}$ **D.** $\frac{\log 8}{\log 20}$

Remember What You Learned

4. Sometimes it is easier to remember a formula if you can state it in words. State the change of base formula in words. **Sample answer: To change the logarithm of a number from one base to another, divide the log of the original number in the old base by the log of the new base in the old base.**

NAME ______________________ DATE __________ PERIOD _____

9-4 Study Guide and Intervention

Common Logarithms

Common Logarithms Base 10 logarithms are called **common logarithms**. The expression $\log_{10} x$ is usually written without the subscript as $\log x$. Use the LOG key on your calculator to evaluate common logarithms.

The relation between exponents and logarithms gives the following identity.

Inverse Property of Logarithms and Exponents	$10^{\log x} = x$

Example 1 **Evaluate log 50 to four decimal places.**

Use the LOG key on your calculator. To four decimal places, log 50 = 1.6990.

Example 2 **Solve $3^{2x+1} = 12$.**

$3^{2x+1} = 12$ — Original equation

$\log 3^{2x+1} = \log 12$ — Property of Equality for Logarithms

$(2x + 1) \log 3 = \log 12$ — Power Property of Logarithms

$2x + 1 = \frac{\log 12}{\log 3}$ — Divide each side by log 3.

$2x = \frac{\log 12}{\log 3} - 1$ — Subtract 1 from each side.

$x = \left(\frac{1}{2}\ \frac{\log 12}{\log 3}\ 1\right)$ — Multiply each side by $\frac{1}{2}$.

$x \approx 0.6309$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. log 18 **1.2553**
2. log 39 **1.5911**
3. log 120 **2.0792**
4. log 5.8 **0.7634**
5. log 42.3 **1.6263**
6. log 0.003 **−2.5229**

Solve each equation or inequality. Round to four decimal places.

7. $4^{3x} = 12$ **0.5975**
8. $6^{x+2} = 18$ **−0.3869**
9. $5^{4x-2} = 120$ **1.2437**
10. $7^{3x-1} \geq 21$ **$\{x \mid x \geq 0.8549\}$**
11. $2.4^{x+4} = 30$ **−0.1150**
12. $6.5^{2x} \geq 200$ **$\{x \mid x \geq 1.4153\}$**
13. $3.6^{4x-1} = 85.4$ **1.1180**
14. $2^{x+5} = 3^{x-2}$ **13.9666**
15. $9^{3x} = 4^{5x+2}$ **−8.1595**
16. $6^{x-5} = 2^{7x+3}$ **−3.6069**

NAME ______________________ DATE __________ PERIOD _____

9-4 Study Guide and Intervention *(continued)*

Common Logarithms

Change of Base Formula The following formula is used to change expressions with different logarithmic bases to common logarithm expressions.

Change of Base Formula	For all positive numbers a, b, and n, where $a \neq 1$ and $b \neq 1$, $\log_a n = \frac{\log_b n}{\log_b a}$

Example **Express $\log_8 15$ in terms of common logarithms. Then approximate its value to four decimal places.**

$\log_8 15 = \frac{\log_{10} 15}{\log_{10} 8}$ — Change of Base Formula

≈ 1.3023 — Simplify.

The value of $\log_8 15$ is approximately 1.3023.

Exercises

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

1. $\log_3 16$ **$\frac{\log 16}{\log 3}$, 2.5237**
2. $\log_2 40$ **$\frac{\log 40}{\log 2}$, 5.3219**
3. $\log_5 35$ **$\frac{\log 35}{\log 5}$, 2.2091**
4. $\log_4 22$ **$\frac{\log 22}{\log 4}$, 2.2297**
5. $\log_{12} 200$ **$\frac{\log 200}{\log 12}$, 2.1322**
6. $\log_2 50$ **$\frac{\log 50}{\log 2}$, 5.6439**
7. $\log_5 0.4$ **$\frac{\log 0.4}{\log 5}$, −0.5693**
8. $\log_3 2$ **$\frac{\log 2}{\log 3}$, 0.6309**
9. $\log_4 28.5$ **$\frac{\log 28.5}{\log 4}$, 2.4164**
10. $\log_3 (20)^2$ **$\frac{2 \log 20}{\log 3}$, 5.4537**
11. $\log_6 (5)^4$ **$\frac{4 \log 5}{\log 6}$, 3.5930**
12. $\log_8 (4)^5$ **$\frac{5 \log 4}{\log 8}$, 3.3333**
13. $\log_5 (8)^3$ **$\frac{3 \log 8}{\log 5}$, 3.8761**
14. $\log_2 (3.6)^6$ **$\frac{6 \log 3.6}{\log 2}$, 11.0880**
15. $\log_{12} (10.5)^4$ **$\frac{4 \log 10.5}{\log 12}$, 3.7851**
16. $\log_3 \sqrt{150}$ **$\frac{\log 150}{2 \log 3}$, 2.2804**
17. $\log_4 \sqrt[3]{39}$ **$\frac{\log 39}{3 \log 4}$, 0.8809**
18. $\log_5 \sqrt[4]{1600}$ **$\frac{\log 1600}{4 \log 5}$, 1.1460**

NAME ______________________ DATE __________ PERIOD _____

9-4 Skills Practice

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. log 6 **0.7782**
2. log 15 **1.1761**
3. log 1.1 **0.0414**
4. log 0.3 **−0.5229**

Use the formula pH = −log[H+] to find the pH of each substance given its concentration of hydrogen ions.

5. gastric juices: [H+] = 1.0×10^{-1} mole per liter **1.0**
6. tomato juice: [H+] = 7.94×10^{-5} mole per liter **4.1**
7. blood: [H+] = 3.98×10^{-8} mole per liter **7.4**
8. toothpaste: [H+] = 1.26×10^{-10} mole per liter **9.9**

Solve each equation or inequality. Round to four decimal places.

9. $3^x > 243$ **$\{x \mid x > 5\}$**
10. $16^v \le \frac{1}{4}$ **$\left\{v \,\middle|\, v \le -\frac{1}{2}\right\}$**
11. $8^p = 50$ **1.8813**
12. $7^y = 15$ **1.3917**
13. $5^{3b} = 106$ **0.9659**
14. $4^{5k} = 37$ **0.5209**
15. $12^{7p} = 120$ **0.2752**
16. $9^{2m} = 27$ **0.75**
17. $3^{r-5} = 4.1$ **6.2843**
18. $8^{y+4} > 15$ **$\{y \mid y > -2.6977\}$**
19. $7.6^{d+3} = 57.2$ **−1.0048**
20. $0.5^{t-8} = 16.3$ **3.9732**
21. $42^{x^2} = 84$ **±1.0888**
22. $5^{x^2+1} = 10$ **±0.6563**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_3 7$ **$\frac{\log_{10} 7}{\log_{10} 3}$; 1.7712**
24. $\log_5 66$ **$\frac{\log_{10} 66}{\log_{10} 5}$; 2.6032**
25. $\log_2 35$ **$\frac{\log_{10} 35}{\log_{10} 2}$; 5.1293**
26. $\log_6 10$ **$\frac{\log_{10} 10}{\log_{10} 6}$; 1.2851**

NAME ______________________ DATE __________ PERIOD _____

9-4 Practice

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. log 101 **2.0043**
2. log 2.2 **0.3424**
3. log 0.05 **−1.3010**

Use the formula pH = −log[H+] to find the pH of each substance given its concentration of hydrogen ions.

4. milk: [H+] = 2.51×10^{-7} mole per liter **6.6**
5. acid rain: [H+] = 2.51×10^{-6} mole per liter **5.6**
6. black coffee: [H+] = 1.0×10^{-5} mole per liter **5.0**
7. milk of magnesia: [H+] = 3.16×10^{-11} mole per liter **10.5**

Solve each equation or inequality. Round to four decimal places.

8. $2^x < 25$ **$\{x \mid x < 4.6439\}$**
9. $5^a = 120$ **2.9746**
10. $6^z = 45.6$ **2.1319**
11. $9^m \ge 100$ **$\{m \mid m \ge 2.0959\}$**
12. $3.5^x = 47.9$ **3.0885**
13. $8.2^y = 64.5$ **1.9802**
14. $2^{b+1} \le 7.31$ **$\{b \mid b \le 1.8699\}$**
15. $4^{2x} = 27$ **1.1887**
16. $2^{a-4} = 82.1$ **10.3593**
17. $9^{z-2} > 38$ **$\{z \mid z > 3.6555\}$**
18. $5^{w+3} = 17$ **−1.2396**
19. $30^{x^2} = 50$ **±1.0725**
20. $5^{x^2-3} = 72$ **±2.3785**
21. $4^{2x} = 9^{x+1}$ **3.8188**
22. $2^{n+1} = 5^{2n-1}$ **0.9117**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_5 12$ **$\frac{\log_{10} 12}{\log_{10} 5}$; 1.5440**
24. $\log_8 32$ **$\frac{\log_{10} 32}{\log_{10} 8}$; 1.6667**
25. $\log_{11} 9$ **$\frac{\log_{10} 9}{\log_{10} 11}$; 0.9163**
26. $\log_2 18$ **$\frac{\log_{10} 18}{\log_{10} 2}$; 4.1699**
27. $\log_9 6$ **$\frac{\log_{10} 6}{\log_{10} 9}$; 0.8155**
28. $\log_7 \sqrt{8}$ **$\frac{\log_{10} 8}{2 \log_{10} 7}$; 0.5343**

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions [H+] in the soil is not less than 1.58×10^{-8} mole per liter. What is the pH of the soil in which these irises will flourish? **7.8 or less**

30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk? **about 5000**

31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria? **about 5.6 h**

32. **SOUND** An equation for loudness L in decibels is given by $L = 10 \log R$, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise? **1000**

Lesson 9-4

NAME ______________________ DATE ____________ PERIOD _____

9-4 Word Problem Practice

Common Logarithms

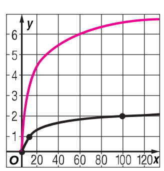
1. OTHER BASES Jamie needs to figure out what $\log_2 3$ is, but she only has a table of common logarithms. In the table, she finds that $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.4771$. Using this information, to the nearest thousandth, what is $\log_2 3$?
1.585

2. PH The pH of a solution is given by

$$-\log_{10} C,$$

where C is the concentration of hydrogen ions in moles per liter. A solution of baking soda creates a hydrogen ion concentration 5×10^{-9} of mole per liter. What is the pH of a solution of baking soda? Round your answer to the nearest tenth.
8.3

3. GRAPHING The graph of $y = \log_{10} x$ is shown below. Use the fact that $\frac{1}{\log_{10} 2} \approx 3.32$ to sketch a graph of $y = \log_2 x$ on the same graph.



4. SCIENTIFIC NOTATION When a number n is written in scientific notation, it has the form $n = s \times 10^p$, where s is a number greater than or equal to 1 and less than 10 and p is an integer. Show that $p \le \log_{10} n < p + 1$.

$n = s \times 10^p$
$\log_{10} n = \log_{10} (s \times 10^p)$
$= \log_{10} s + \log_{10} 10^p$
$= \log_{10} s + p$
Because $1 \le s < 10$,
$0 \le \log_{10} s < 1$.
Therefore, $p \le \log_{10} n < p + 1$.

LOG TABLE For Exercises 5 and 6, use the following information.

Marjorie is looking through some old science books owned by her grandfather. At the back of one of them, there is a table of logarithms base 10. However, the book is worn out and some of the entries are unreadable.

Table of Common Logarithms (4 decimal places of accuracy)	
x	$\log_{10} x$
2	0.3010
3	0.4771
4	?
5	0.6989
6	?

5. Approximately what are the missing entries in the table? Round off your answers to the nearest thousandth.
$\log_{10} 4 \approx 0.602$
$\log_{10} 6 \approx 0.778$

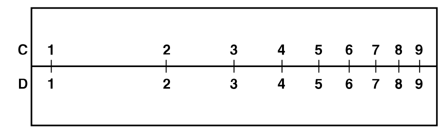
6. How can you use this table to determine $\log_{10} 1.5$?
Sample answer:
$\log_{10} 1.5$
$= \log_{10} 3 + \log_{10} 5 - \log_{10} 10$
$\approx 0.4771 + 0.6989 - 1$
$= 0.1760$

NAME ______________________ DATE ____________ PERIOD _____

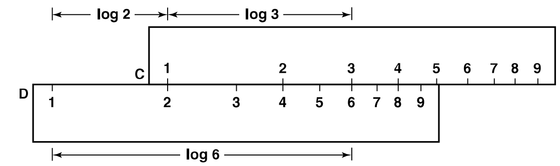
9-4 Enrichment

The Slide Rule

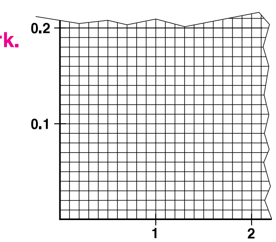
Before the invention of electronic calculators, computations were often performed on a slide rule. A slide rule is based on the idea of logarithms. It has two movable rods labeled with C and D scales. Each of the scales is logarithmic.



To multiply 2×3 on a slide rule, move the C rod to the right as shown below. You can find 2×3 by adding log 2 to log 3, and the slide rule adds the lengths for you. The distance you get is 0.778, or the logarithm of 6.

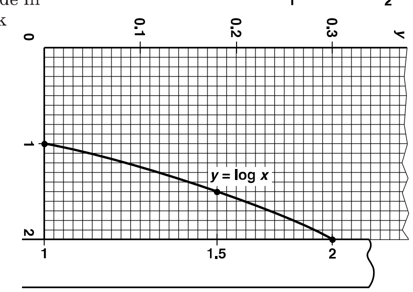


Follow the steps to make a slide rule. **1–2. See students' work.**



1. Use graph paper that has small squares, such as 10 squares to the inch. Using the scales shown at the right, plot the curve $y = \log x$ for $x = 1, 1.5,$ and the whole numbers from 2 through 10. Make an obvious heavy dot for each point plotted.

2. You will need two strips of cardboard. A 5-by-7 index card, cut in half the long way, will work fine. Turn the graph you made in Exercise 1 sideways and use it to mark a logarithmic scale on each of the two strips. The figure shows the mark for 2 being drawn.



3. Explain how to use a slide rule to divide 8 by 2. **Line up the 2 on the C scale with the 8 on the D scale. The quotient is the number on the D scale below the 1 on the C scale.**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Lesson Reading Guide

Base e and Natural Logarithms

Get Ready for the Lesson

Read the introduction to Lesson 9-5 in your textbook.

Suppose that you deposit $675 in a savings account that pays an annual interest rate of 5%. In each case listed below, indicate which method of compounding would result in more money in your account at the end of one year.

a. annual compounding or monthly compounding **monthly**

b. quarterly compounding or daily compounding **daily**

c. daily compounding or continuous compounding **continuous**

Read the Lesson

1. Jagdish entered the following keystrokes in his calculator:

LN 5) ENTER

The calculator returned the result 1.609437912. Which of the following conclusions are correct? **d and f**

a. The common logarithm of 5 is about 1.6094.

b. The natural logarithm of 5 is exactly 1.609437912.

c. The base 5 logarithm of e is about 1.6094.

d. The natural logarithm of 5 is about 1.609438.

e. $10^{1.609437912}$ is very close to 5.

f. $e^{1.609437912}$ is very close to 5.

2. Match each expression from the first column with its value in the second column. Some choices may be used more than once or not at all.

a. $e^{\ln 5}$ **IV** — **I.** 1

b. $\ln 1$ **V** — **II.** 10

c. $e^{\ln e}$ **VI** — **III.** −1

d. $\ln e^5$ **IV** — **IV.** 5

e. $\ln e$ **I** — **V.** 0

f. $\ln\left(\frac{1}{e}\right)$ **III** — **VI.** e

Remember What You Learned

3. A good way to remember something is to explain it to someone else. Suppose that you are studying with a classmate who is puzzled when asked to evaluate $\ln e^3$. How would you explain to him an easy way to figure this out? **Sample answer: *ln* means *natural log*. The natural log of e^3 is the power to which you raise e to get e^3. This is obviously 3.**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention

Base e and Natural Logarithms

Base e and Natural Logarithms The irrational number $e \approx 2.71828\ldots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

Natural Base e	As n increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\ldots$. $\ln x = \log_e x$

The functions $y = e^x$ and $y = \ln x$ are inverse functions.

Inverse Property of Base e and Natural Logarithms	$e^{\ln x} = x$ $\quad \ln e^x = x$

Natural base expressions can be evaluated using the e^x and ln keys on your calculator.

Example 1 **Evaluate ln 1685.**

Use a calculator.
$\ln 1685 \approx 7.4295$

Example 2 **Write a logarithmic equation equivalent to $e^{2x} = 7$.**

$e^{2x} = 7 \rightarrow \log_e 7 = 2x$ or $2x = \ln 7$

Example 3 **Evaluate $\ln e^{18}$.**

Use the Inverse Property of Base e and Natural Logarithms.
$\ln e^{18} = 18$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. ln 732 **6.5958**
2. ln 84,350 **11.3427**
3. ln 0.735 **−0.3079**
4. ln 100 **4.6052**
5. ln 0.0824 **−2.4962**
6. ln 2.388 **0.8705**
7. ln 128,245 **11.7617**
8. ln 0.00614 **−5.0929**

Write an equivalent exponential or logarithmic equation.

9. $e^{15} = x$ **$\ln x = 15$**
10. $e^{3x} = 45$ **$3x = \ln 45$**
11. $\ln 20 = x$ **$e^x = 20$**
12. $\ln x = 8$ **$x = e^8$**
13. $e^{-5x} = 0.2$ **$-5x = \ln 0.2$**
14. $\ln (4x) = 9.6$ **$4x = e^{9.6}$**
15. $e^{8.2} = 10x$ **$\ln 10x = 8.2$**
16. $\ln 0.0002 = x$ **$e^x = 0.0002$**

Evaluate each expression.

17. $\ln e^3$ **3**
18. $e^{\ln 42}$ **42**
19. $e^{\ln 0.5}$ **0.5**
20. $\ln e^{16.2}$ **16.2**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention *(continued)*

Base e and Natural Logarithms

Equations and Inequalities with *e* and ln All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

Example **Solve each equation or inequality.**

a. $3e^{2x} + 2 = 10$

$3e^{2x} + 2 = 10$	Original equation
$3e^{2x} = 8$	Subtract 2 from each side.
$e^{2x} = \frac{8}{3}$	Divide each side by 3.
$\ln e^{2x} = \ln \frac{8}{3}$	Property of Equality for Logarithms
$2x = \ln \frac{8}{3}$	Inverse Property of Exponents and Logarithms
$x = \frac{1}{2} \ln \frac{8}{3}$	Multiply each side by $\frac{1}{2}$.
$x \approx 0.4904$	Use a calculator.

b. $\ln (4x - 1) < 2$

$\ln (4x - 1) < 2$	Original inequality
$e^{\ln (4x - 1)} < e^2$	Write each side using exponents and base *e*.
$0 < 4x - 1 < e^2$	Inverse Property of Exponents and Logarithms
$1 < 4x < e^2 + 1$	Addition Property of Inequalities
$\frac{1}{4} < x < \frac{1}{4}(e^2 + 1)$	Multiplication Property of Inequalities
$0.25 < x < 2.0973$	Use a calculator.

Exercises

Solve each equation or inequality.

1. $e^{4x} = 120$ **1.1969**

2. $e^x \le 25$ **$\{x | x \le 3.2189\}$**

3. $e^{x-2} + 4 = 21$ **4.8332**

4. $\ln 6x \ge 4$ **$x \ge 9.0997$**

5. $\ln (x + 3) - 5 = -2$ **17.0855**

6. $e^{-8x} \le 50$ **$\{x | x \ge -0.4890\}$**

7. $e^{4x-1} - 3 = 12$ **0.9270**

8. $\ln (5x + 3) = 3.6$ **6.7196**

9. $2e^{3x} + 5 = 2$ **no solution**

10. $6 + 3e^{x+1} = 21$ **0.6094**

11. $\ln (2x - 5) = 8$ **1492.9790**

12. $\ln 5x + \ln 3x > 9$ **$\{x | x > 23.2423\}$**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Skills Practice

Base e and Natural Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. e^3 **20.0855**

2. e^{-2} **0.1353**

3. $\ln 2$ **0.6931**

4. $\ln 0.09$ **−2.4079**

Write an equivalent exponential or logarithmic equation.

5. $e^x = 3$ **$x = \ln 3$**

6. $e^4 = 8x$ **$4 = \ln 8x$**

7. $\ln 15 = x$ **$e^x = 15$**

8. $\ln x \approx 0.6931$ **$x \approx e^{0.6931}$**

Evaluate each expression.

9. $e^{\ln 3}$ **3**

10. $e^{\ln 2x}$ **$2x$**

11. $\ln e^{-2.5}$ **−2.5**

12. $\ln e^y$ **y**

Solve each equation or inequality.

13. $e^x \ge 5$ **$\{x | x \ge 1.6094\}$**

14. $e^x < 3.2$ **$\{x | x < 1.1632\}$**

15. $2e^x - 1 = 11$ **1.7918**

16. $5e^x + 3 = 18$ **1.0986**

17. $e^{3x} = 30$ **1.1337**

18. $e^{-4x} > 10$ **$\{x | x < -0.5756\}$**

19. $e^{5x} + 4 > 34$ **$\{x | x > 0.6802\}$**

20. $1 - 2e^{2x} = -19$ **1.1513**

21. $\ln 3x = 2$ **2.4630**

22. $\ln 8x = 3$ **2.5107**

23. $\ln (x - 2) = 2$ **9.3891**

24. $\ln (x + 3) = 1$ **−0.2817**

25. $\ln (x + 3) = 4$ **51.5982**

26. $\ln x + \ln 2x = 2$ **1.9221**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Practice

Base e and Natural Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. $e^{1.5}$ **4.4817**
2. ln 8 **2.0794**
3. ln 3.2 **1.1632**
4. $e^{-0.6}$ **0.5488**
5. $e^{4.2}$ **66.6863**
6. ln 1 **0**
7. $e^{-2.5}$ **0.0821**
8. ln 0.037 **−3.2968**

Write an equivalent exponential or logarithmic equation.

9. $\ln 50 = x$ — $e^x = 50$
10. $\ln 36 = 2x$ — $e^{2x} = 36$
11. $\ln 6 \approx 1.7918$ — $e^{1.7918} \approx 6$
12. $\ln 9.3 \approx 2.2300$ — $e^{2.2300} \approx 9.3$
13. $e^x = 8$ — $x = \ln 8$
14. $e^5 = 10x$ — $5 = \ln 10x$
15. $e^{-x} = 4$ — $x = -\ln 4$
16. $e^2 = x + 1$ — $2 = \ln (x + 1)$

Evaluate each expression.

17. $e^{\ln 12}$ **12**
18. $e^{\ln 3x}$ **3x**
19. $\ln e^{-1}$ **−1**
20. $\ln e^{-2y}$ **−2y**

Solve each equation or inequality.

21. $e^x < 9$ — $\{x \mid x < 2.1972\}$
22. $e^{-x} = 31$ — **−3.4340**
23. $e^x = 1.1$ — **0.0953**
24. $e^x = 5.8$ — **1.7579**
25. $2e^x - 3 = 1$ — **0.6931**
26. $5e^x + 1 \geq 7$ — $\{x \mid x \geq 0.1823\}$
27. $4 + e^x = 19$ — **2.7081**
28. $-3e^x + 10 < 8$ — $\{x \mid x > -0.4055\}$
29. $e^{3x} = 8$ — **0.6931**
30. $e^{-4x} = 5$ — **−0.4024**
31. $e^{0.5x} = 6$ — **3.5835**
32. $2e^{5x} = 24$ — **0.4970**
33. $e^{2x} + 1 = 55$ — **1.9945**
34. $e^{3x} - 5 = 32$ — **1.2036**
35. $9 + e^{2x} = 10$ — **0**
36. $e^{-3x} + 7 \geq 15$ — $\{x \mid x \leq -0.6931\}$
37. $\ln 4x = 3$ — **5.0214**
38. $\ln (-2x) = 7$ — **−548.3166**
39. $\ln 2.5x = 10$ — **8810.5863**
40. $\ln (x - 6) = 1$ — **8.7183**
41. $\ln (x + 2) = 3$ — **18.0855**
42. $\ln (x + 3) = 5$ — **145.4132**
43. $\ln 3x + \ln 2x = 9$ — **36.7493**
44. $\ln 5x + \ln x = 7$ — **14.8097**

INVESTING For Exercises 45 and 46, use the formula for continuously compounded interest, $A = Pe^{rt}$, where P is the principal, r is the annual interest rate, and t is the time in years.

45. If Sarita deposits \$1000 in an account paying 3.4% annual interest compounded continuously, what is the balance in the account after 5 years? **\$1185.30**

46. How long will it take the balance in Sarita's account to reach \$2000? **about 20.4 yr**

47. **RADIOACTIVE DECAY** The amount of a radioactive substance y that remains after t years is given by the equation $y = ae^{kt}$, where a is the initial amount present and k is the decay constant for the radioactive substance. If $a = 100$, $y = 50$, and $k = -0.035$, find t. **about 19.8 yr**

NAME ______________________ DATE ____________ PERIOD _____

9-5 Word Problem Practice

Base e and Natural Logarithms

1. **INTEREST** Horatio opens a bank account that pays 2.3% annual interest compounded continuously. He makes an initial deposit of 10,000. What will be the balance of the account in 10 years? Assume that he makes no additional deposits and no withdrawals.
   \$12,586

2. **INTEREST** Janie's bank pays 2.8% annual interest compounded continuously on savings accounts. She placed \$2000 in the account. How long will it take for her initial deposit to double in value? Assume that she makes no additional deposits and no withdrawals. Round your answer to the nearest quarter year.
   24.75 yr

3. **BACTERIA** A bacterial population grows exponentially, doubling every 72 hours. Let P be the initial population size and let t be time in hours. Write a formula using the natural base exponential function that gives the size of the population as a function of P and t.
   $P = e^{\frac{\ln 2}{72}t}$

4. **POPULATION** The equation $A = A_0e^{rt}$ describes the growth of the world's population where A is the population at time t, A_0 is the population at $t = 0$, and r is the annual growth rate. How long will take a population of 6.5 billion to increase to 9 billion if the annual growth rate is 2%?
   16.3 yr

MONEY MANAGEMENT For Exercises 5–7, use the following information.

Linda wants to invest \$20,000. She is looking at two possible accounts. Account A is a standard savings account that pays 3.4% annual interest compounded continuously. Account B would pay her a fixed amount of \$200 every quarter.

5. If Linda can invest the money for 5 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?
   Account B; she'll make \$24000 − \$23706.10 = \$293.90 more

6. If Linda can invest the money for 10 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?
   Account A; she'll make \$28098.95 − \$28000 = \$98.95 more

7. If Linda can invest the money for 20 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?
   Account A; she'll make \$39477.55 − \$36000 = \$3477.55 more

Lesson 9-5

NAME ____________________ DATE __________ PERIOD _____

9-5 Enrichment

Approximations for π and e

The following expression can be used to approximate e. If greater and greater values of n are used, the value of the expression approximates e more and more closely.

$$\left(1 + \frac{1}{n}\right)^n$$

Another way to approximate e is to use this infinite sum. The greater the value of n, the closer the approximation.

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \ldots + \frac{1}{2 \cdot 3 \cdot 4 \cdot \ldots \cdot n} + \ldots$$

In a similar manner, π can be approximated using an infinite product discovered by the English mathematician John Wallis (1616–1703).

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \ldots \cdot \frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \ldots$$

Solve each problem.

1. Use a calculator with an e^x key to find e to 7 decimal places. **2.7182818**

2. Use the expression $\left(1 + \frac{1}{n}\right)^n$ to approximate e to 3 decimal places. Use 5, 100, 500, and 7000 as values of n. **2.488, 2.705, 2.716, 2.718**

3. Use the infinite sum to approximate e to 3 decimal places. Use the whole numbers from 3 through 6 as values of n. **2.667, 2.708, 2.717, 2.718**

4. Which approximation method approaches the value of e more quickly? **the infinite sum**

5. Use a calculator with a π key to find π to 7 decimal places. **3.1415927**

6. Use the infinite product to approximate π to 3 decimal places. Use the whole numbers from 3 through 6 as values of n. **2.926, 2.972, 3.002, 3.023**

7. Does the infinite product give good approximations for π quickly? **no**

8. Show that $\pi^4 + \pi^5$ is equal to e^6 to 4 decimal places. **To 4 decimal places, they both equal 403.4288.**

9. Which is larger, e^π or π^e? **$e^\pi > \pi^e$**

10. The expression x reaches a maximum value at $x = e$. Use this fact to prove the inequality you found in Exercise 9. **$e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$; $\left(e^{\frac{1}{e}}\right)^{\pi e} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi e}$; $e^\pi > \pi^e$**

NAME ____________________ DATE __________ PERIOD _____

9-6 Lesson Reading Guide

Exponential Growth and Decay

Get Ready for the Lesson

Read the introduction to Lesson 9-6 in your textbook.

- Between which two years shown in the table did the car depreciate by the greatest amount? **between years 0 and 1**
- Describe two ways to calculate the value of the car 6 years after it was purchased. (Do not actually calculate the value.) **Sample answer: 1. Multiply $9200.66 by 0.16 and subtract the result from $9200.66. 2. Multiply $9200.66 by 0.84.**

Read the Lesson

1. State whether each situation is an example of exponential *growth* or *decay*.
   a. A city had 42,000 residents in 1980 and 128,000 residents in 2000. **growth**
   b. Raul compared the value of his car when he bought it new to the value when he traded it in six years later. **decay**
   c. A paleontologist compared the amount of carbon-14 in the skeleton of an animal when it died to the amount 300 years later. **decay**
   d. Maria deposited $750 in a savings account paying 4.5% annual interest compounded quarterly. She did not make any withdrawals or further deposits. She compared the balance in her passbook immediately after she opened the account to the balance 3 years later. **growth**

2. State whether each equation represents exponential growth or decay.
   a. $y = 5e^{0.15t}$ **growth**
   b. $y = 1000(1 - 0.05)^t$ **decay**
   c. $y = 0.3e^{-1200t}$ **decay**
   d. $y = 2(1 + 0.0001)^t$ **growth**

Remember What You Learned

3. Visualizing their graphs is often a good way to remember the difference between mathematical equations. How can your knowledge of the graphs of exponential equations from Lesson 9-1 help you to remember that equations of the form $y = a(1 + r)^t$ represent exponential growth, while equations of the form $y = a(1 - r)^t$ represent exponential decay? **Sample answer: If $a > 0$, the graph of $y = ab^x$ is always increasing if $b > 1$ and is always decreasing if $0 < b < 1$. Since r is always a positive number, if $b = 1 + r$, the base will be greater than 1 and the function will be increasing (growth), while if $b = 1 - r$, the base will be less than 1 and the function will be decreasing (decay).**

NAME ______________________ DATE ____________ PERIOD _____

9-6 Study Guide and Intervention

Exponential Growth and Decay

Exponential Decay Depreciation of value and radioactive decay are examples of **exponential decay**. When a quantity decreases by a fixed percent each time period, the amount of the quantity after t time periods is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent decrease expressed as a decimal.

Another exponential decay model often used by scientists is $y = ae^{-kt}$, where k is a constant.

Example **CONSUMER PRICES As technology advances, the price of many technological devices such as scientific calculators and camcorders goes down. One brand of hand-held organizer sells for $89.**

a. If its price decreases by 6% per year, how much will it cost after 5 years?

Use the exponential decay model with initial amount $89, percent decrease 0.06, and time 5 years.

$y = a(1 - r)^t$ — Exponential decay formula

$y = 89(1 - 0.06)^5$ — $a = 89$, $r = 0.06$, $t = 5$

$y = \$65.32$

After 5 years the price will be $65.32.

b. After how many years will its price be $50?

To find when the price will be $50, again use the exponential decay formula and solve for t.

$y = a(1 - r)^t$ — Exponential decay formula

$50 = 89(1 - 0.06)^t$ — $y = 50$, $a = 89$, $r = 0.06$

$\frac{50}{89} = (0.94)^t$ — Divide each side by 89.

$\log\left(\frac{50}{89}\right) = \log(0.94)^t$ — Property of Equality for Logarithms

$\log\left(\frac{50}{89}\right) = t \log 0.94$ — Power Property

$t = \frac{\log\left(\frac{50}{89}\right)}{\log 0.94}$ — Divide each side by log 0.94.

$t \approx 9.3$

The price will be $50 after about 9.3 years.

Exercises

1. **BUSINESS** A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a $500 item drop below $100? **6 weeks**

CARBON DATING Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of Carbon-14, t is the number of years ago the animal lived, and y is the remaining amount after t years.

2. How old is a fossil remain that has lost 95% of its Carbon-14? **about 25,000 years old**

3. How old is a skeleton that has 95% of its Carbon-14 remaining? **about 427.5 years old**

NAME ______________________ DATE ____________ PERIOD _____

9-6 Study Guide and Intervention *(continued)*

Exponential Growth and Decay

Exponential Growth Population increase and growth of bacteria colonies are examples of **exponential growth**. When a quantity increases by a fixed percent each time period, the amount of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is $y = ae^{kt}$, where k is a constant.

Example **A computer engineer is hired for a salary of $28,000. If she gets a 5% raise each year, after how many years will she be making $50,000 or more?**

Use the exponential growth model with $a = 28{,}000$, $y = 50{,}000$, and $r = 0.05$ and solve for t.

$y = a(1 + r)^t$ — Exponential growth formula

$50{,}000 = 28{,}000(1 + 0.05)^t$ — $y = 50{,}000$, $a = 28{,}000$, $r = 0.05$

$\frac{50}{28} = (1.05)^t$ — Divide each side by 28,000.

$\log\left(\frac{50}{28}\right) = \log(1.05)^t$ — Property of Equality of Logarithms

$\log\left(\frac{50}{28}\right) = t \log 1.05$ — Power Property

$t = \frac{\log\left(\frac{50}{28}\right)}{\log 1.05}$ — Divide each side by log 1.05.

$t \approx 11.9$ years — Use a calculator.

If raises are given annually, she will be making over $50,000 in 12 years.

Exercises

1. **BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find k for the growth formula $y = ae^{kt}$, where t is in minutes. **about 0.0175**

2. **INVESTMENT** Carl plans to invest $500 at 8.25% interest, compounded continuously. How long will it take for his money to triple? **about 14 years**

3. **SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full? **about 5 years**

4. **EXERCISE** Hugo begins a walking program by walking $\frac{1}{2}$ mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day? **24 weeks**

5. **VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase? **about 14%**

NAME ______________________ DATE __________ PERIOD _____

9-6 Skills Practice

Exponential Growth and Decay

1. **FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline? **about 8.3 yr**

2. **INVESTING** Alex invests $2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth $3600? **10 yr**

3. **POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula $P = ae^{rt}$, find the expected population of the city in 30 years if the growth rate r of the population is 1.5% per year, a represents the current population in millions, and t represents the time in years. **about 5.5 million**

4. **POPULATION** The population P in thousands of a city can be modeled by the equation $P = 80e^{0.015t}$, where t is the time in years. In how many years will the population of the city be 120,000? **about 27 yr**

5. **BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%? **about 4.9 days**

6. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of k for this element? **about 0.00002845**

7. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for $12,500. The GPS depreciated by a fixed rate of 6% and is now worth $8600. How long ago did Abu buy the GPS system? **about 6.0 yr**

8. **BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$? **about 12.13%**

NAME ______________________ DATE __________ PERIOD _____

9-6 Practice

Exponential Growth and Decay

1. **INVESTING** The formula $A = P\left(1 + \frac{r}{2}\right)^{2t}$ gives the value of an investment after t years in an account that earns an annual interest rate r compounded twice a year. Suppose $500 is invested at 6% annual interest compounded twice a year. In how many years will the investment be worth $1000? **about 11.7 yr**

2. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000 if the growth rate per hour is 85%? **about 7.5 h**

3. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant k in the decay formula for the substance. **about 0.02166**

4. **DEPRECIATION** A piece of machinery valued at $250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to $100,000? **about 7.2 yr**

5. **INFLATION** For Dave to buy a new car comparably equipped to the one he bought 8 years ago would cost $12,500. Since Dave bought the car, the inflation rate for cars like his has been at an average annual rate of 5.1%. If Dave originally paid $8400 for the car, how long ago did he buy it? **about 8 yr**

6. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt-60, is radioactive and has a half-life of 5.7 years. Cobalt-60 is used to trace the path of nonradioactive substances in a system. What is the value of k for Cobalt-60? **about 0.1216**

7. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use $k = 0.00012$. **about 50,000 yr**

8. **POPULATION** The population of rabbits in an area is modeled by the growth equation $P(t) = 8e^{0.26t}$, where P is in thousands and t is in years. How long will it take for the population to reach 25,000? **about 4.4 yr**

9. **DEPRECIATION** A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally $12,000, in how many months is it worth $7350? **about 12 mo**

10. **BIOLOGY** In a laboratory, a culture increases from 30 to 195 organisms in 5 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$? **about 45.4%**