NAME ______________________ DATE ____________ PERIOD _____

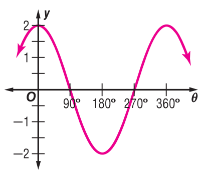
14-1 Skills Practice

Graphing Trigonometric Functions

Find the amplitude, if it exists, and period of each function. Then graph each function.

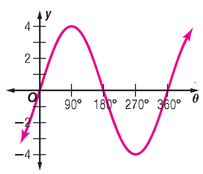
1. $y = 2 \cos \theta$

2; 360°



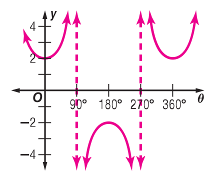
2. $y = 4 \sin \theta$

4; 360°



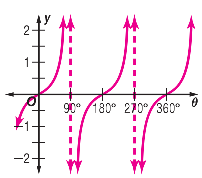
3. $y = 2 \sec \theta$

no amplitude; 360°



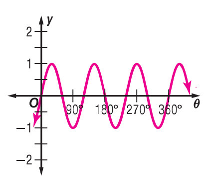
4. $y = \frac{1}{2} \tan \theta$

no amplitude; 180°



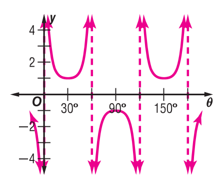
5. $y = \sin 3\theta$

1; 120°



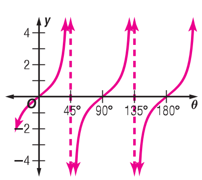
6. $y = \csc 3\theta$

no amplitude; 120°



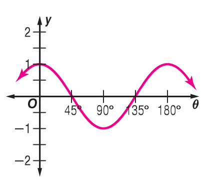
7. $y = \tan 2\theta$

no amplitude; 90°



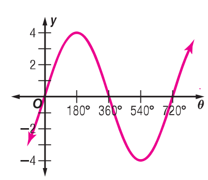
8. $y = \cos 2\theta$

1; 180°



9. $y = 4 \sin \frac{1}{2}\theta$

4; 720°



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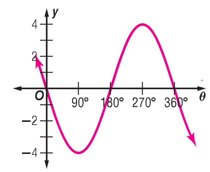
14-1 Practice (Average)

Graphing Trigonometric Functions

Find the amplitude, if it exists, and period of each function. Then graph each function.

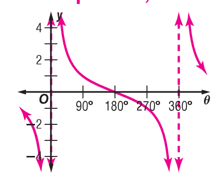
1. $y = -4 \sin \theta$

4; 360°



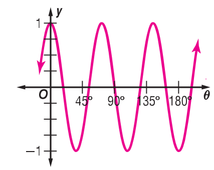
2. $y = \cot \frac{1}{2}\theta$

no amplitude; 360°



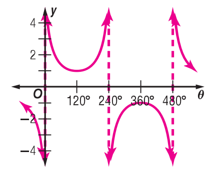
3. $y = \cos 5\theta$

1; 72°



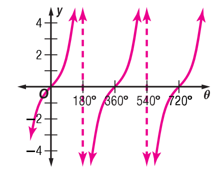
4. $y = \csc \frac{3}{4}\theta$

no amplitude; 480°

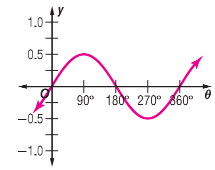


5. $y = 2 \tan \frac{1}{2}\theta$

no amplitude; 360°

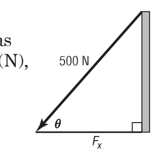


6. $2y = \sin \theta$ **$\frac{1}{2}$; 360°**



FORCE For Exercises 7 and 8, use the following information.

An anchoring cable exerts a force of 500 Newtons on a pole. The force has the horizontal and vertical components F_x and F_y. (A force of one Newton (N), is the force that gives an acceleration of 1 m/sec^2 to a mass of 1 kg.)



7. The function $F_x = 500 \cos \theta$ describes the relationship between the angle θ and the horizontal force. What are the amplitude and period of this function? **500; 360°**

8. The function $F_y = 500 \sin \theta$ describes the relationship between the angle θ and the vertical force. What are the amplitude and period of this function? **500; 360°**

WEATHER For Exercises 9 and 10, use the following information.

The function $y = 60 + 25 \sin \frac{\pi}{6}t$, where t is in months and $t = 0$ corresponds to April 15, models the average high temperature in degrees Fahrenheit in Centerville.

9. Determine the period of this function. What does this period represent? **12; a calendar year**

10. What is the maximum high temperature and when does this occur? **85°F; July 15**

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14-1 Word Problem Practice

Graphing Trigonometric Functions

PHYSICS For Exercises 1–3, use the following information.

The following chart gives functions which model the wave patterns of different colors of light emitted from a particular source, where y is the height of the wave in nanometers and t is the length from the start of the wave in nanometers.

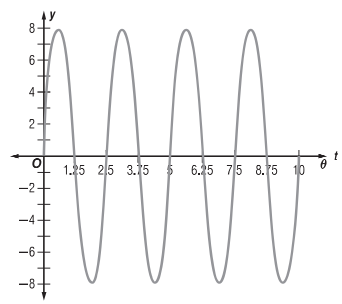
Color	Function
Red	$y = 300 \sin\left(\frac{\pi}{350}t\right)$
Orange	$y = 125 \sin\left(\frac{\pi}{305}t\right)$
Yellow	$y = 460 \sin\left(\frac{\pi}{290}t\right)$
Green	$y = 200 \sin\left(\frac{\pi}{260}t\right)$
Blue	$y = 40 \sin\left(\frac{\pi}{235}t\right)$
Violet	$y = 80 \sin\left(\frac{\pi}{210}t\right)$

1. What are the amplitude and period of the function describing green light waves? **200 nm, 520 nm**

2. The intensity of a light wave corresponds directly to its amplitude. Which color emitted from the source is the most intense? **yellow**

3. The color of light depends on the period of the wave. Which color has the shortest period? The longest period? **violet, red**

4. **SWIMMING** As Charles swims a 25 meter sprint, the position of his right hand relative to the water surface can be modeled by the graph below, where h is the height of the had in inches from the water level and t is the seconds past the start of the sprint. What function describes this graph? $y = 8 \sin\left(\frac{4\pi}{5}t\right)$



ENVIRONMENT For Exercise 5 and 6, use the following information.

In a certain forest, the leaf density can be modeled by the equation $y = 20 + 15 \sin\left(\frac{\pi}{6}(t - 3)\right)$ where y represents the number of leaves per square foot and t represents the number of months after January.

5. Determine the period of this function. What does this period represent? **The period of the function is 12. This represents one full year.**

6. What is the maximum leaf density that occurs in this forest and when does this occur? **35 leaves per square foot, June**

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14-1 Enrichment

Blueprints

Interpreting blueprints requires the ability to select and use trigonometric functions and geometric properties. The figure below represents a plan for an improvement to a roof. The metal fitting shown makes a 30° angle with the horizontal. The vertices of the geometric shapes are *not* labeled in these plans. Relevant information must be selected and the appropriate function used to find the unknown measures.

Example **Find the unknown measures in the figure at the right.**

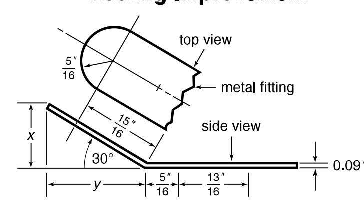
The measures x and y are the legs of a right triangle.

The measure of the hypotenuse is $\frac{15}{16}$ in. + $\frac{5}{16}$ in. or $\frac{20}{16}$ in.

$\frac{y}{\frac{20}{16}} = \cos 30°$ $\qquad$ $\frac{x}{\frac{20}{16}} = \sin 30°$

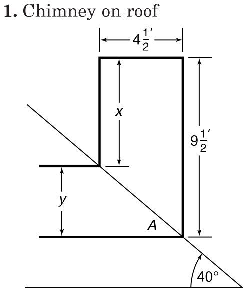
$y = 1.08$ in. $\qquad$ $x = 0.63$ in.

Roofing Improvement



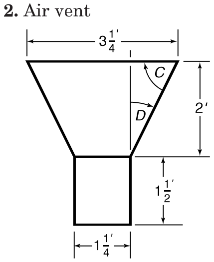
Find the unknown measures of each of the following.

1. Chimney on roof



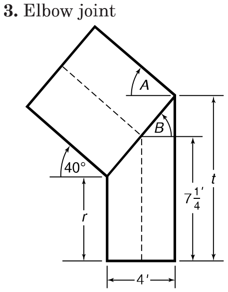
y = 3.78′
x = 5.72′
∠A = 40°

2. Air vent



∠C = 63.43°
∠D = 26.57°

3. Elbow joint



∠A = 40°
∠B = 50°
t = 9.63′
r = 4.87′

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14-2 Lesson Reading Guide

Translations of Trigonometric Graphs

Pre-Activity **How can translations of trigonometric graphs be used to show animal populations?**

Read the introduction to Lesson 14-2 at the top of page 769 in your textbook.

According to the model given in your textbook, what would be the estimated rabbit population for January 1, 2005? **1200**

Reading the Lesson

1. Determine whether the graph of each function represents a shift of the parent function *to the left*, *to the right*, *upward*, or *downward*. (Do not actually graph the functions.)

a. $y = \sin(\theta + 90°)$ **to the left**

b. $y = \sin\theta + 3$ **upward**

c. $y = \cos\left(\theta - \frac{\pi}{3}\right)$ **to the right**

d. $y = \tan\theta - 4$ **downward**

2. Determine whether the graph of each function has an *amplitude change*, *period change*, *phase shift*, or *vertical shift* compared to the graph of the parent function. (More than one of these may apply to each function. Do not actually graph the functions.)

a. $y = 3\sin\left(\theta + \frac{5\pi}{6}\right)$ **amplitude change and phase shift**

b. $y = \cos(2\theta + 70°)$ **period change and phase shift**

c. $y = -4\cos 3\theta$ **amplitude change and period change**

d. $y = \sec\frac{1}{2}\theta + 3$ **period change and vertical shift**

e. $y = \tan\left(\theta - \frac{\pi}{4}\right) - 1$ **phase shift and vertical shift**

f. $y = 2\sin\left(\frac{1}{3}\theta + \frac{\pi}{6}\right) - 4$ **amplitude change, period change, phase shift, and vertical shift**

Helping You Remember

3. Many students have trouble remembering which of the functions $y = \sin(\theta + \alpha)$ and $y = \sin(\theta - \alpha)$ represents a shift to the left and which represents a shift to the right. Using $\alpha = 45°$, explain a good way to remember which is which.

Sample answer: Although sine curves are infinitely repeating periodic graphs, think of $y = \sin x$ starting a period or cycle at (0, 0). Then $y = \sin(\theta + 45°)$ "starts early" at (−45°), a shift of 45° to the left, while $y = \sin(\theta - 45°)$ "starts late" at 45°, a shift of 45° to the right.

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14-2 Study Guide and Intervention

Translations of Trigonometric Graphs

Horizontal Translations When a constant is subtracted from the angle measure in a trigonometric function, a **phase shift** of the graph results.

Phase Shift	The horizontal phase shift of the graphs of the functions $y = a\sin b(\theta - h)$, $y = a\cos b(\theta - h)$, and $y = a\tan b(\theta - h)$ is h, where $b > 0$. If $h > 0$, the shift is to the right. If $h < 0$, the shift is to the left.

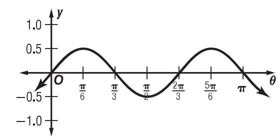
Example **State the amplitude, period, and phase shift for $y = \frac{1}{2}\cos 3\left(\theta - \frac{\pi}{2}\right)$. Then graph the function.**

Amplitude: $a = \left|\frac{1}{2}\right|$ or $\frac{1}{2}$

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$

Phase Shift: $h = \frac{\pi}{2}$

The phase shift is to the right since $\frac{\pi}{2} > 0$.

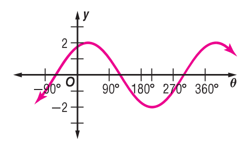


Exercises

State the amplitude, period, and phase shift for each function. Then graph the function.

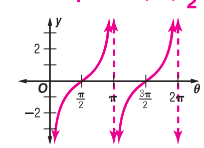
1. $y = 2\sin(\theta + 60°)$

2; 360°; 60° to the left



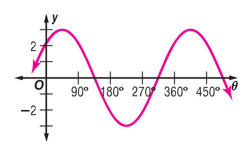
2. $y = \tan\left(\theta - \frac{\pi}{2}\right)$

no amplitude; π; $\frac{\pi}{2}$ to the right



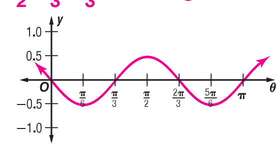
3. $y = 3\cos(\theta - 45°)$

3; 360°; 45° to the right



4. $y = \frac{1}{2}\sin 3\left(\theta - \frac{\pi}{3}\right)$

$\frac{1}{2}$; $\frac{2\pi}{3}$; $\frac{\pi}{3}$ to the right



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14-2 Study Guide and Intervention *(continued)*

Translations of Trigonometric Graphs

Vertical Translations When a constant is added to a trigonometric function, the graph is shifted vertically.

Vertical Shift	The vertical shift of the graphs of the functions $y = a \sin b(\theta - h) + k$, $y = a \cos b(\theta - h) + k$, and $y = a \tan b(\theta - h) + k$ is k. If $k > 0$, the shift is up. If $k < 0$, the shift is down.

The **midline** of a vertical shift is $y = k$.

Graphing Trigonometric Functions	**Step 1** Determine the vertical shift, and graph the midline. **Step 2** Determine the amplitude, if it exists. Use dashed lines to indicate the maximum and minimum values of the function. **Step 3** Determine the period of the function and graph the appropriate function. **Step 4** Determine the phase shift and translate the graph accordingly.

Example **State the vertical shift, equation of the midline, amplitude, and period for $y = \cos 2\theta - 3$. Then graph the function.**

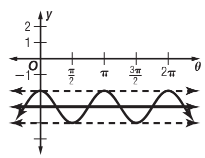
Vertical Shift: $k = -3$, so the vertical shift is 3 units down.

The equation of the midline is $y = -3$.

Amplitude: $|a| = |1|$ or 1

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|2|}$ or π

Since the amplitude of the function is 1, draw dashed lines parallel to the midline that are 1 unit above and below the midline. Then draw the cosine curve, adjusted to have a period of π.

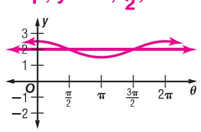


Exercises

State the vertical shift, equation of the midline, amplitude, and period for each function. Then graph the function.

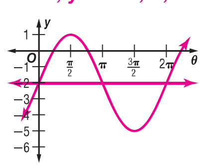
1. $y = \frac{1}{2} \cos \theta + 2$

2 up; $y = 2$; $\frac{1}{2}$; 2π



2. $y = 3 \sin \theta - 2$

2 down; $y = -2$; 3; 2π



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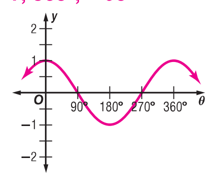
14-2 Skills Practice

Translations of Trigonometric Graphs

State the amplitude, period, and phase shift for each function. Then graph the function.

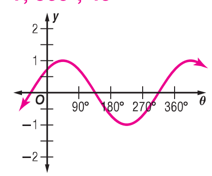
1. $y = \sin(\theta + 90°)$

1; 360°; −90°



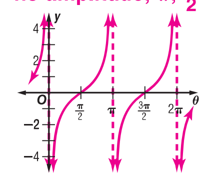
2. $y = \cos(\theta - 45°)$

1; 360°; 45°



3. $y = \tan\left(\theta - \frac{\pi}{2}\right)$

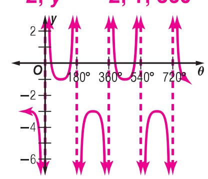
no amplitude; π; $\frac{\pi}{2}$



State the vertical shift, equation of the midline, amplitude, and period for each function. Then graph the function.

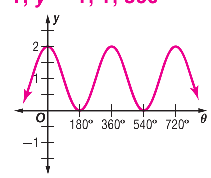
4. $y = \csc \theta - 2$

−2; $y = -2$; 1; 360°

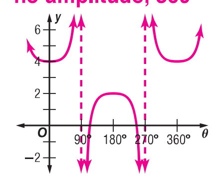


5. $y = \cos \theta + 1$

1; $y = 1$; 1; 360°



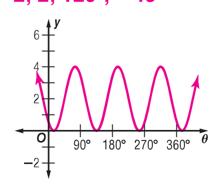
6. $y = \sec \theta + 3$ **3; $y = 3$; no amplitude; 360°**



State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.

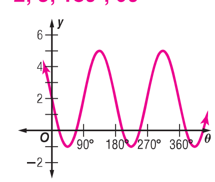
7. $y = 2 \cos [3(\theta + 45°)] + 2$

2; 2; 120°; −45°



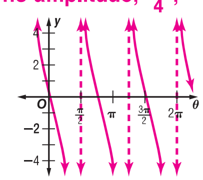
8. $y = 3 \sin [2(\theta - 90°)] + 2$

2; 3; 180°; 90°



9. $y = 4 \cot \left[\frac{4}{3}\left(\theta + \frac{\pi}{4}\right)\right] - 2$

−2; no amplitude; $\frac{3\pi}{4}$; $-\frac{\pi}{4}$



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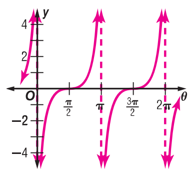
14-2 Practice (Average)

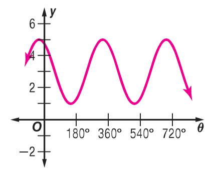
Translations of Trigonometric Graphs

State the vertical shift, amplitude, period, and phase shift for each function. Then graph the function.

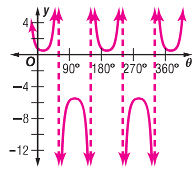
1. $y = \frac{1}{2}\tan\left(\theta - \frac{\pi}{2}\right)$
no vertical shift; no amplitude; π; $\frac{\pi}{2}$



2. $y = 2\cos(\theta + 30°) + 3$
3; 2; 360; −30°



3. $y = 3\csc(2\theta + 60°) - 2.5$
−2.5; no amplitude; 180°; −60°



ECOLOGY For Exercises 4–6, use the following information.

The population of an insect species in a stand of trees follows the growth cycle of a particular tree species. The insect population can be modeled by the function $y = 40 + 30\sin 6t$, where t is the number of years since the stand was first cut in November, 1920.

4. How often does the insect population reach its maximum level? **every 60 yr**

5. When did the population last reach its maximum? **1995**

6. What condition in the stand do you think corresponds with a minimum insect population? **Sample answer: The species on which the insect feeds has been cut.**

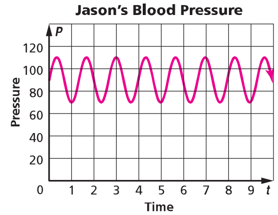
BLOOD PRESSURE For Exercises 7–9, use the following information.

Jason's blood pressure is 110 over 70, meaning that the pressure oscillates between a maximum of 110 and a minimum of 70. Jason's heart rate is 45 beats per minute. The function that represents Jason's blood pressure P can be modeled using a sine function with no phase shift.

7. Find the amplitude, midline, and period in seconds of the function. **20; $P = 90$; $1\frac{1}{3}$ s**

8. Write a function that represents Jason's blood pressure P after t seconds. **$P = 20\sin 270t + 90$**

9. Graph the function.



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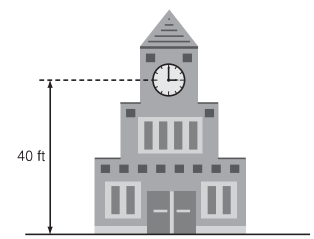
14-2 Word Problem Practice

Translations of Trigonometric Graphs

CLOCKS For Exercises 1–4, use the following information.

A town hall has a tower with a clock on its face. The center of the clock is 40 feet above street level. The minute hand of the clock has a radius of four feet.

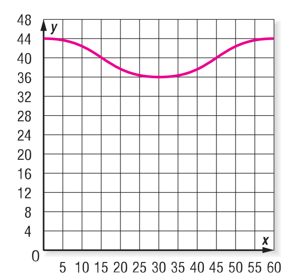


1. What is the maximum height of the tip of the minute had above street level?
44 feet

2. What is the minimum height of the tip of the minute hand above street level?
36 feet

3. Write a sine function that represents the height above street level of the tip of the minute hand for t minutes after midnight.
$y = \sin\left(\frac{\pi}{30}t + \frac{\pi}{2}\right) + 40$?

4. Graph the function from our answer to Exercise 3.



ANIMAL POPULATION For Exercises 1–4, use the following information.

The population of predators and pry in a closed ecological system tends to vary periodically over time. In a certain system, the population of snakes S can be represented by $S = 100 + 20\sin\left(\frac{\pi}{5}t\right)$, where t is the number of years since January 1, 2008. In that same system, the population of rats can be represented by $R = 200 + 75\sin\left(\frac{\pi}{5}t = \frac{\pi}{10}\right)$.

5. What is the maximum snake population?
120

6. When is this population first reached?
2.5 yr

7. What is the minimum rat population?
125

8. When is this population first reached?
7 yr

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14-2 Enrichment

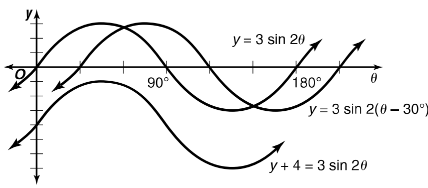
Translating Graphs of Trigonometric Functions

Three graphs are shown at the right:

$y = 3 \sin 2\theta$

$y = 3 \sin 2(\theta - 30°)$

$y + 4 = 3 \sin 2\theta$



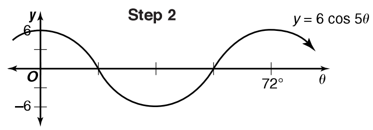
Replacing θ with $(\theta - 30°)$ translates the graph to the right. Replacing y with $y + 4$ translates the graph 4 units down.

Example **Graph one cycle of $y = 6 \cos (5\theta + 80°) + 2$.**

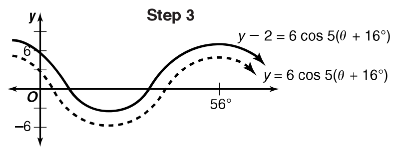
Step 1 Transform the equation into the form $y - k = a \cos b(\theta - h)$.

$y - 2 = 6 \cos 5(\theta + 16°)$

Step 2 Sketch $y = 6 \cos 5\theta$.



Step 3 Translate $y = 6 \cos 5\theta$ to obtain the desired graph.



Sketch these graphs on the same coordinate system. **See students' graphs.**

1. $y = 3 \sin 2(\theta + 45°)$ **2.** $y - 1 = 3 \sin 2\theta$ **3.** $y + 5 = 3 \sin 2(\theta + 90°)$

On another piece of paper, graph one cycle of each curve. **See students' graphs.**

4. $y = 2 \sin 4(\theta - 50°)$ **5.** $y = 5 \sin (3\theta + 90°)$

6. $y = 6 \cos (4\theta + 360°) + 3$ **7.** $y = 6 \cos 4\theta + 3$

8. The graphs for problems 6 and 7 should be the same. Use the sum formula for cosine of a sum to show that the equations are equivalent.

$\cos (4\theta + 360°) = (\cos 4\theta)(\cos 360°) - (\sin 4\theta)(\sin 360°)$
$= (\cos 4\theta)(1) - (\sin 4\theta)(0)$
$= \cos 4\theta$

So, $y = 6 \cos (4\theta + 360°) + 3$ and $y = 6 \cos 4\theta + 3$ are equivalent.

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14-3 Lesson Reading Guide

Trigonometric Identities

Pre-Activity **How can trigonometry be used to model the path of a baseball?**

Read the introduction to Lesson 14-3 at the top of page 777 in your textbook.

Suppose that a baseball is hit from home plate with an initial velocity of 58 feet per second at an angle of 36° with the horizontal from an initial height of 5 feet. Show the equation that you would use to find the height of the ball 10 seconds after the ball is hit. (Show the formula with the appropriate numbers substituted, but do not do any calculations.)

$h = \left(\frac{-16}{58^2 \cos^2 36°}\right) 10^2 + \left(\frac{\sin 36°}{\cos 36°}\right) 10 + 5$

Reading the Lesson

1. Match each expression from the list on the left with an expression from the list on the right that is equal to it for all values for which each expression is defined. (Some of the expressions from the list on the right may be used more than once or not at all.)

a. $\sec^2 \theta - \tan^2 \theta$ **iii**	**i.** $\frac{1}{\sin \theta}$
b. $\cot^2 \theta + 1$ **v**	**ii.** $\tan \theta$
c. $\frac{\sin \theta}{\cos \theta}$ **ii**	**iii.** 1
d. $\sin^2 \theta + \cos^2 \theta$ **iii**	**iv.** $\sec \theta$
e. $\csc \theta$ **i**	**v.** $\csc^2 \theta$
f. $\frac{1}{\cos \theta}$ **iv**	**vi.** $\cot \theta$
g. $\frac{\cos \theta}{\sin \theta}$ **vi**	

2. Write an identity that you could use to find each of the indicated trigonometric values and tell whether that value is positive or negative. (Do not actually find the values.)

a. $\tan \theta$, if $\sin \theta = -\frac{4}{5}$ and $180° < \theta < 270°$ **$\tan \theta = \frac{\sin \theta}{\cos \theta}$; positive**

b. $\sec \theta$, if $\tan \theta = -3$ and $90° < \theta < 180°$ **$\tan^2 \theta + 1 = \sec^2 \theta$; negative**

Helping You Remember

3. A good way to remember something new is to relate it to something you already know. How can you use the unit circle definitions of the sine and cosine that you learned in Chapter 13 to help you remember the Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$?
Sample answer: On a unit circle, $x = \cos \theta$ and $y = \sin \theta$. The equation of the unit circle is $x^2 + y^2 = 1$, so this is equivalent to the equation $\cos^2 \theta + \sin^2 \theta = 1$.

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14-3 Study Guide and Intervention

Trigonometric Identities

Find Trigonometric Values A **trigonometric identity** is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

Basic Trigonometric Identities	Quotient Identities	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	
	Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
	Pythagorean Identities	$\cos^2\theta + \sin^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$\cot^2\theta + 1 = \csc^2\theta$

Example **Find the value of cot θ if $\csc\theta = -\frac{11}{5}$; $180° < \theta < 270°$.**

$\cot^2\theta + 1 = \csc^2\theta$ — Trigonometric identity

$\cot^2\theta + 1 = \left(-\frac{11}{5}\right)^2$ — Substitute $-\frac{11}{5}$ for csc θ.

$\cot^2\theta + 1 = \frac{121}{25}$ — Square $-\frac{11}{5}$.

$\cot^2\theta = \frac{96}{25}$ — Subtract 1 from each side.

$\cot\theta = \pm\frac{4\sqrt{6}}{5}$ — Take the square root of each side.

Since θ is in the third quadrant, cot θ is positive, Thus $\cot\theta = \frac{4\sqrt{6}}{5}$.

Exercises

Find the value of each expression.

1. tan θ, if cot θ = 4; $180° < \theta < 270°$ $\frac{1}{4}$
2. csc θ, if $\cos\theta = \frac{\sqrt{3}}{2}$; $0° \le \theta < 90°$ 2
3. cos θ, if $\sin\theta = \frac{3}{5}$; $0° \le \theta < 90°$ $\frac{4}{5}$
4. sec θ, if $\sin\theta = \frac{1}{3}$; $0° \le \theta < 90°$ $\frac{3\sqrt{2}}{4}$
5. cos θ, if $\tan\theta = -\frac{4}{3}$; $90° < \theta < 180°$ $-\frac{3}{5}$
6. tan θ, if $\sin\theta = \frac{3}{7}$; $0° \le \theta < 90°$ $\frac{3\sqrt{10}}{20}$
7. sec θ, if $\cos\theta = -\frac{7}{8}$; $90° < \theta < 180°$ $-\frac{8}{7}$
8. sin θ, if $\cos\theta = \frac{6}{7}$; $270° \le \theta < 360°$ $-\frac{\sqrt{13}}{7}$
9. cot θ, if $\csc\theta = \frac{12}{5}$; $90° < \theta < 180°$ $-\frac{\sqrt{119}}{5}$
10. sin θ, if $\csc\theta = -\frac{9}{4}$; $270° < \theta < 360°$ $-\frac{4}{9}$

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14-3 Study Guide and Intervention *(continued)*

Trigonometric Identities

Simplify Expressions The simplified form of a trigonometric expression is written as a numerical value or in terms of a single trigonometric function, if possible. Any of the trigonometric identities on page 849 can be used to simplify expressions containing trigonometric functions.

Example 1 **Simplify $(1 - \cos^2\theta)\sec\theta\cot\theta + \tan\theta\sec\theta\cos^2\theta$.**

$$(1 - \cos^2\theta)\sec\theta\cot\theta + \tan\theta\sec\theta\cos^2\theta = \sin^2\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} \cdot \cos^2\theta$$
$$= \sin\theta + \sin\theta$$
$$= 2\sin\theta$$

Example 2 **Simplify $\frac{\sec\theta \cdot \cot\theta}{1 - \sin\theta} - \frac{\csc\theta}{1 + \sin\theta}$.**

$$\frac{\sec\theta \cdot \cot\theta}{1 - \sin\theta} - \frac{\csc\theta}{1 + \sin\theta} = \frac{\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}}{1 - \sin\theta} - \frac{\frac{1}{\sin\theta}}{1 + \sin\theta}$$
$$= \frac{\frac{1}{\sin\theta}(1 + \sin\theta) - \frac{1}{\sin\theta}(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$$
$$= \frac{\frac{1}{\sin\theta} + 1 - \frac{1}{\sin\theta} + 1}{1 - \sin^2\theta}$$
$$= \frac{2}{\cos^2\theta}$$

Exercises

Simplify each expression.

1. $\frac{\tan\theta \cdot \csc\theta}{\sec\theta}$ 1
2. $\frac{\sin\theta \cdot \cot\theta}{\sec^2\theta - \tan^2\theta}$ cos θ
3. $\frac{\sin^2\theta - \cot\theta \cdot \tan\theta}{\cot\theta \cdot \sin\theta}$ −cos θ
4. $\frac{\cos\theta}{\sec\theta - \tan\theta}$ 1 + sin θ
5. $\frac{\tan\theta \cdot \cos\theta}{\sin\theta} + \cot\theta \cdot \sin\theta \cdot \tan\theta \cdot \csc\theta$ 2
6. $\frac{\csc^2\theta - \cot^2\theta}{\tan\theta \cdot \cos\theta}$ csc θ
7. $3\tan\theta \cdot \cot\theta + 4\sin\theta \cdot \csc\theta + 2\cos\theta \cdot \sec\theta$ 9
8. $\frac{1 - \cos^2\theta}{\tan\theta \cdot \sin\theta}$ cos θ

Lesson 14-3

Answers

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14-3 Skills Practice

Trigonometric Identities

Find the value of each expression.

1. $\sin\theta$, if $\cos\theta = -\frac{4}{5}$ and $90° < \theta < 180°$ **$\frac{3}{5}$**

2. $\cos\theta$, if $\tan\theta = 1$ and $180° < \theta < 270°$ **$-\frac{\sqrt{2}}{2}$**

3. $\sec\theta$, if $\tan\theta = 1$ and $0° \le \theta < 90°$ **$\sqrt{2}$**

4. $\cos\theta$, if $\tan\theta = \frac{1}{2}$ and $0° \le \theta < 90°$ **$\frac{2\sqrt{5}}{5}$**

5. $\tan\theta$, if $\sin\theta = -\frac{\sqrt{2}}{2}$ and $180° < \theta < 270°$ **1**

6. $\cos\theta$, if $\sec\theta = 2$ and $270° < \theta < 360°$ **$\frac{1}{2}$**

7. $\cos\theta$, if $\csc\theta = -2$ and $180° < \theta < 270°$ **$-\frac{\sqrt{3}}{2}$**

8. $\tan\theta$, if $\cos\theta = -\frac{2\sqrt{5}}{5}$ and $180° < \theta < 270°$ **$\frac{1}{2}$**

9. $\cos\theta$, if $\cot\theta = -\frac{3}{2}$ and $90° < \theta < 180°$ **$-\frac{3\sqrt{13}}{13}$**

10. $\csc\theta$, if $\cos\theta = \frac{8}{17}$ and $0° < \theta < 90°$ **$\frac{17}{15}$**

11. $\cot\theta$, if $\csc\theta = -2$ and $180° < \theta < 270°$ **$\sqrt{3}$**

12. $\tan\theta$, if $\sin\theta = -\frac{5}{13}$ and $180° < \theta < 270°$ **$\frac{5}{12}$**

Simplify each expression.

13. $\sin\theta\sec\theta$ **$\tan\theta$**

14. $\csc\theta\sin\theta$ **1**

15. $\cot\theta\sec\theta$ **$\csc\theta$**

16. $\frac{\cos\theta}{\sec\theta}$ **$\cos^2\theta$**

17. $\tan\theta + \cot\theta$ **$\frac{1}{\cos\theta\sin\theta}$**

18. $\csc\theta\tan\theta - \tan\theta\sin\theta$ **$\cos\theta$**

19. $\frac{1-\sin^2\theta}{\sin\theta + 1}$ **$1 - \sin\theta$**

20. $\csc\theta + \cot\theta$ **$\frac{1+\cos\theta}{\sin\theta}$**

21. $\frac{\sin^2\theta + \cos^2\theta}{1-\cos^2\theta}$ **$\csc^2\theta$**

22. $1 + \frac{\tan^2\theta}{1+\sec\theta}$ **$\sec\theta$**

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14-3 Practice (Average)

Trigonometric Identities

Find the value of each expression.

1. $\sin\theta$, if $\cos\theta = \frac{5}{13}$ and $0° \le \theta < 90°$ **$\frac{12}{13}$**

2. $\sec\theta$, if $\sin\theta = -\frac{15}{17}$ and $180° < \theta < 270°$ **$-\frac{17}{8}$**

3. $\cot\theta$, if $\cos\theta = \frac{3}{10}$ and $270° < \theta < 360°$ **$-\frac{3\sqrt{91}}{91}$**

4. $\sin\theta$, if $\cot\theta = \frac{1}{2}$ and $0° \le \theta < 90°$ **$\frac{2\sqrt{5}}{5}$**

5. $\cot\theta$, if $\csc\theta = -\frac{3}{2}$ and $180° < \theta < 270°$ **$\frac{\sqrt{5}}{2}$**

6. $\sec\theta$, if $\csc\theta = -8$ and $270° < \theta < 360°$ **$\frac{8\sqrt{7}}{21}$**

7. $\sec\theta$, if $\tan\theta = 4$ and $180° < \theta < 270°$ **$-\sqrt{17}$**

8. $\sin\theta$, if $\tan\theta = -\frac{1}{2}$ and $270° < \theta < 360°$ **$-\frac{\sqrt{5}}{5}$**

9. $\cot\theta$, if $\tan\theta = \frac{2}{5}$ and $0° \le \theta < 90°$ **$\frac{5}{2}$**

10. $\cot\theta$, if $\cos\theta = \frac{1}{3}$ and $270° < \theta < 360°$ **$-\frac{\sqrt{2}}{4}$**

Simplify each expression.

11. $\csc\theta\tan\theta$ **$\sec x$**

12. $\frac{\sin^2\theta}{\tan^2\theta}$ **$\cos^2\theta$**

13. $\sin^2\theta\cot^2\theta$ **$\cos^2\theta$**

14. $\cot^2\theta + 1$ **$\csc^2\theta$**

15. $\frac{\csc^2\theta - \cot^2\theta}{1-\cos^2\theta}$ **$\csc^2\theta$**

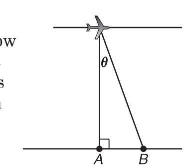
16. $\frac{\csc\theta - \sin\theta}{\cos\theta}$ **$\cot\theta$**

17. $\sin\theta + \cos\theta\cot\theta$ **$\csc\theta$**

18. $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta}$ **$2\tan\theta$**

19. $\sec^2\theta\cos^2\theta - \tan^2\theta$ **$\sec^2\theta$**

20. **AERIAL PHOTOGRAPHY** The illustration shows a plane taking an aerial photograph of point A. Because the point is directly below the plane, there is no distortion in the image. For any point B not directly below the plane, however, the increase in distance creates distortion in the photograph. This is because as the distance from the camera to the point being photographed increases, the exposure of the film reduces by $(\sin\theta)(\csc\theta - \sin\theta)$. Express $(\sin\theta)(\csc\theta - \sin\theta)$ in terms of $\cos\theta$ only. **$\cos^2\theta$**



21. **TSUNAMIS** The equation $y = a\sin\theta t$ represents the height of the waves passing a buoy at a time t in seconds. Express a in terms of $\csc\theta t$. **$a = y\csc\theta t$**

Lesson 14-3

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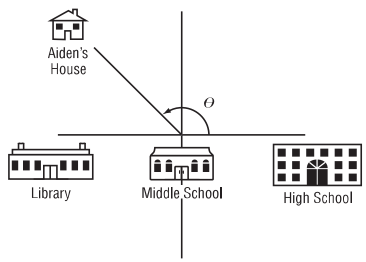
14-3 Word Problem Practice

Trigonometric Identities

MAPS For Exercises 1–3, use the following information.

The figure below shows a map of some buildings in Aiden's hometown. The sine of the angle θ formed by the high school, the middle school, and Aiden's house is $\frac{3}{7}$.



1. What is the cosine of the angle?

$\frac{2\sqrt{10}}{7}$

2. What is the tangent of the angle?

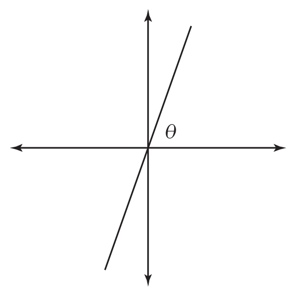
$\frac{3\sqrt{10}}{20}$

3. What are the sine, cosine, and tangent of the angle formed by the library, the middle school, and Aiden's house?

$\frac{3}{7}, \frac{2\sqrt{10}}{7}, \frac{3\sqrt{10}}{20}$

GEOMETRY For Exercises 4–6, use the following information.

When a line is drawn in the coordinate plane, the tangent of the angle that the line makes with the horizontal axis is equal to the slope of the line (for non-vertical lines).



The cosine of the angle θ the line in the figure makes with the horizontal is $\frac{1}{3}$.

4. Explain two ways to determine the slope of the line. Draw a representative triangle with the length of the side adjacent to angle θ equal to 1 and the hypotenuse equal to 3. **Use the Pythagorean Theorem to find the length of the other leg. Compute the tangent from the definition. Or use the given value of cosine of θ and the Pythagorean Identity to compute sine of θ. Finally, use the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$.**

5. Compute the sine and tangent of the angle.

$\frac{2\sqrt{2}}{3}, 2\sqrt{2}$

6. What is the slope of the line? $2\sqrt{2}$

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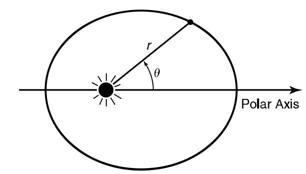
14-3 Enrichment

Planetary Orbits

The orbit of a planet around the sun is an ellipse with the sun at one focus. Let the pole of a polar coordinate system be that focus and the polar axis be toward the other focus. The polar equation of an ellipse is



$r = \frac{2ep}{1 - e\cos\theta}$. Since $2p = \frac{b^2}{c}$ and $b^2 = a^2 - c^2$,

$2p = \frac{a^2 - c^2}{c} = \frac{a^2}{c}\left(1 - \frac{c^2}{a^2}\right)$. Because $e = \frac{c}{a}$,

$2p = a\left(\frac{a}{c}\right)\left(1 - \left(\frac{c}{a}\right)^2\right) = a\left(\frac{1}{e}\right)(1 - e^2)$.

Therefore, $2ep = a(1 - e^2)$. Substituting into the polar equation of an ellipse yields an equation that is useful for finding distances from the planet to the sun.

$r = \frac{a(1 - e^2)}{1 - e\cos\theta}$

Note that e is the eccentricity of the orbit and a is the length of the semi-major axis of the ellipse. Also, a is the mean distance of the planet from the sun.

Example **The mean distance of Venus from the sun is 67.24×10^6 miles and the eccentricity of its orbit is .006788. Find the minimum and maximum distances of Venus from the sun.**

The minimum distance occurs when $\theta = \pi$.

$r = \frac{67.24 \times 10^6(1 - 0.006788^2)}{1 - 0.006788 \cos \pi} = 66.78 \times 10^6$ miles

The maximum distance occurs when $\theta = 0$.

$r = \frac{67.24 \times 10^6(1 - 0.006788^2)}{1 - 0.006788 \cos 0} = 67.70 \times 10^6$ miles

Complete each of the following.

1. The mean distance of Mars from the sun is 141.64×10^6 miles and the eccentricity of its orbit is 0.093382. Find the minimum and maximum distances of Mars from the sun.

max. distance = 15.49×10^7 mi; min. distance = 12.84×10^7 mi

2. The minimum distance of Earth from the sun is 91.445×10^6 miles, and the eccentricity of its orbit is 0.016734. Find the mean and maximum distances of Earth from the sun.

max. distance = 93.00×10^6 mi; mean distance = 91.47×10^6 mi

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14-4 Lesson Reading Guide

Verifying Trigonometric Identities

Pre-Activity **How can you verify trigonometric identities?**

Read the introduction to Lesson 14-4 at the top of page 782 in your textbook.

For $\theta = -\pi$, 0, or π, $\sin \theta = \sin 2\theta$. Does this mean that $\sin \theta = \sin 2\theta$ is an identity? Explain your reasoning. **Sample answer: No; an identity is an equation that is true for *all* values of a variable for which the functions involved are defined, not just *some* values. If $\theta = \frac{\pi}{4}$, $\sin \theta = \frac{\sqrt{2}}{2}$, and $\sin 2\theta = 1$.**

Reading the Lesson

1. Determine whether each equation is an *identity* or *not an identity*.

   a. $\frac{1}{\sin^2 \theta} - \frac{1}{\tan^2 \theta} = 1$ **identity**

   b. $\frac{\cos \theta}{\sin \theta \tan \theta}$ **not an identity**

   c. $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \cos \theta \sin \theta$ **not an identity**

   d. $\cos^2 \theta (\tan^2 \theta + 1) = 1$ **identity**

   e. $\frac{\sin^2 \theta}{\cos^2 \theta} + \sin \theta \csc \theta = \sec^2 \theta$ **identity**

   f. $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \cos^2 \theta$ **not an identity**

   g. $\tan^2 \theta \cos^2 \theta = \frac{1}{\csc^2 \theta}$ **identity**

   h. $\frac{\sin \theta}{\sec \theta} = \frac{1}{\tan \theta} + \frac{1}{\cot \theta}$ **not an identity**

2. Which of the following is *not* permitted when verifying an identity? **B**

   A. simplifying one side of the identity to match the other side

   B. cross multiplying if the identity is a proportion

   C. simplifying each side of the identity separately to get the same expression on both sides

Helping You Remember

3. Many students have trouble knowing where to start in verifying a trigonometric identity. What is a simple rule that you can remember that you can always use if you don't see a quicker approach? **Sample answer: Write both sides in terms of sines and cosines. Then simplify each side as much as possible.**

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14-4 Study Guide and Intervention

Verifying Trigonometric Identities

Transform One Side of an Equation Use the basic trigonometric identities along with the definitions of the trigonometric functions to verify trigonometric identities. Often it is easier to begin with the more complicated side of the equation and transform that expression into the form of the simpler side.

Example **Verify that each of the following is an identity.**

a. $\frac{\sin \theta}{\cot \theta} - \sec \theta = -\cos \theta$

Transform the left side.

$$\frac{\sin \theta}{\cot \theta} - \sec \theta \stackrel{?}{=} -\cos \theta$$
$$\frac{\sin \theta}{\frac{\cos \theta}{\sin \theta}} - \frac{1}{\cos \theta} \stackrel{?}{=} -\cos \theta$$
$$\frac{\sin^2 \theta}{\cos \theta} - \frac{1}{\cos \theta} \stackrel{?}{=} -\cos \theta$$
$$\frac{\sin^2 - 1}{\cos \theta} \stackrel{?}{=} -\cos \theta$$
$$\frac{-\cos^2 \theta}{\cos \theta} \stackrel{?}{=} -\cos \theta$$
$$-\cos \theta = -\cos \theta$$

b. $\frac{\tan \theta}{\csc \theta} + \cos \theta = \sec \theta$

Transform the left side.

$$\frac{\tan \theta}{\csc \theta} + \cos \theta \stackrel{?}{=} \sec \theta$$
$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\sin \theta}} + \cos \theta \stackrel{?}{=} \sec \theta$$
$$\frac{\sin^2 \theta}{\cos \theta} + \cos \theta \stackrel{?}{=} \sec \theta$$
$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \stackrel{?}{=} \sec \theta$$
$$\frac{1}{\cos \theta} \stackrel{?}{=} \sec \theta$$
$$\sec \theta = \sec \theta$$

Exercises

Verify that each of the following is an identity.

1. $1 + \csc^2 \theta \cdot \cos^2 \theta = \csc^2 \theta$

$$1 + \frac{1}{\sin^2 \theta} \cdot \cos^2 \theta \stackrel{?}{=} \csc^2 \theta$$
$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \stackrel{?}{=} \csc^2 \theta$$
$$\frac{1}{\sin^2 \theta} \stackrel{?}{=} \csc^2 \theta$$
$$\csc^2 \theta = \csc^2 \theta$$

2. $\frac{\sin \theta}{1 - \cos \theta} - \frac{\cot \theta}{1 + \cos \theta} = \frac{1 - \cos^3 \theta}{\sin^3 \theta}$

$$\frac{\sin \theta(1 + \cos \theta) - \frac{\cos \theta}{\sin \theta}(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)} \stackrel{?}{=} \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$
$$\frac{\sin \theta + \sin \theta \cdot \cos \theta - \frac{\cos \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin \theta}}{1 - \cos^2 \theta} \stackrel{?}{=} \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$
$$\frac{\frac{\sin^2 \theta + \sin^2 \theta \cos \theta - \cos \theta + \cos^2 \theta}{\sin \theta}}{\sin^2 \theta} \stackrel{?}{=} \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$
$$\frac{\sin^2 \theta + \cos^2 \theta + \cos \theta(\sin^2 \theta - 1)}{\sin^3 \theta} \stackrel{?}{=} \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$
$$\frac{1 + \cos \theta(-\cos^2 \theta)}{\sin^3 \theta} \stackrel{?}{=} \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$
$$\frac{1 - \cos^3 \theta}{\sin^3 \theta} = \frac{1 - \cos^3 \theta}{\sin^3 \theta}$$

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14-4 Study Guide and Intervention *(continued)*

Verifying Trigonometric Identities

Transform Both Sides of an Equation The following techniques can be helpful in verifying trigonometric identities.

- Substitute one or more basic identities to simplify an expression.
- Factor or multiply to simplify an expression.
- Multiply both numerator and denominator by the same trigonometric expression.
- Write each side of the identity in terms of sine and cosine only. Then simplify each side.

Example **Verify that $\frac{\tan^2\theta + 1}{\sin\theta \cdot \tan\theta \cdot \sec\theta + 1} = \sec^2\theta - \tan^2\theta$ is an identity.**

$$\frac{\tan^2\theta + 1}{\sin\theta \cdot \tan\theta \cdot \sec\theta + 1} \overset{?}{=} \sec^2\theta - \tan^2\theta$$

$$\frac{\sec^2\theta}{\sin\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} + 1} \overset{?}{=} \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} + 1} \overset{?}{=} \frac{1 - \sin^2\theta}{\cos^2\theta}$$

$$\frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}} \overset{?}{=} \frac{\cos^2\theta}{\cos^2\theta}$$

$$\frac{1}{\sin^2\theta + \cos^2\theta} \overset{?}{=} 1$$

$$1 = 1$$

Exercises

Verify that each of the following is an identity.

1. $\csc\theta \cdot \sec\theta = \cot\theta + \tan\theta$

$$\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} \overset{?}{=} \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\sin\theta \cdot \cos\theta} \overset{?}{=} \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos\theta}$$

$$\frac{1}{\sin\theta \cdot \cos\theta} = \frac{1}{\sin\theta \cdot \cos\theta}$$

2. $\frac{\tan^2\theta}{1 - \cos^2\theta} = \frac{\sec\theta}{\cos\theta}$

$$\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\sin^2\theta} \overset{?}{=} \frac{\frac{1}{\cos\theta}}{\cos\theta}$$

$$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

3. $\frac{\cos\theta \cdot \cot\theta}{\sin\theta} = \frac{\csc\theta}{\sin\theta \cdot \sec^2\theta}$

$$\frac{\cos\theta \cdot \frac{\cos\theta}{\sin\theta}}{\sin\theta} \overset{?}{=} \frac{\frac{1}{\sin\theta}}{\sin\theta \cdot \frac{1}{\cos^2\theta}}$$

$$\frac{\cos^2\theta}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

4. $\frac{\csc^2\theta - \cot^2\theta}{\sec^2\theta} = \cot^2\theta(1 - \cos^2\theta)$

$$\frac{\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\cos^2\theta}} \overset{?}{=} \frac{\cos^2\theta}{\sin^2\theta}(\sin^2\theta)$$

$$\cos^2\theta\left(\frac{1 - \cos^2\theta}{\sin^2\theta}\right) \overset{?}{=} \cos^2\theta$$

$$\cos^2\theta\left(\frac{\sin^2\theta}{\sin^2\theta}\right) \overset{?}{=} \cos^2\theta$$

$$\cos^2\theta = \cos^2\theta$$

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14-4 Skills Practice

Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. $\tan\theta\cos\theta = \sin\theta$

$$\tan\theta\cos\theta \overset{?}{=} \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} \cdot \cos\theta \overset{?}{=} \sin\theta$$

$$\sin\theta = \sin\theta$$

2. $\cot\theta\tan\theta = 1$

$$\cot\theta\tan\theta \overset{?}{=} 1$$

$$\frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} \overset{?}{=} 1$$

$$1 = 1$$

3. $\csc\theta\cos\theta = \cot\theta$

$$\csc\theta\cos\theta \overset{?}{=} \cot\theta$$

$$\frac{1}{\sin\theta} \cdot \cos\theta \overset{?}{=} \cot\theta$$

$$\frac{\cos\theta}{\sin\theta} \overset{?}{=} \cot\theta$$

$$\cot\theta = \cot\theta$$

4. $\frac{1 - \sin^2\theta}{\cos\theta} = \cos\theta$

$$\frac{1 - \sin^2\theta}{\cos\theta} \overset{?}{=} \cos\theta$$

$$\frac{\cos^2\theta}{\cos\theta} \overset{?}{=} \cos\theta$$

$$\cos\theta = \cos\theta$$

5. $(\tan\theta)(1 - \sin^2\theta) = \sin\theta\cos\theta$

$$(\tan\theta)(1 - \sin^2\theta) \overset{?}{=} \sin\theta\cos\theta$$

$$\tan\theta\cos^2\theta \overset{?}{=} \sin\theta\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} \cdot \cos^2\theta \overset{?}{=} \sin\theta\cos\theta$$

$$\sin\theta\cos\theta = \sin\theta\cos\theta$$

6. $\frac{\csc\theta}{\sec\theta} = \cot\theta$

$$\frac{\csc\theta}{\sec\theta} \overset{?}{=} \cot\theta$$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} \overset{?}{=} \cot\theta$$

$$\frac{\cos\theta}{\sin\theta} \overset{?}{=} \cot\theta$$

$$\cot\theta = \cot\theta$$

7. $\frac{\sin^2\theta}{1 - \sin^2\theta} = \tan^2\theta$

$$\frac{\sin^2\theta}{1 - \sin^2\theta} \overset{?}{=} \tan^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} \overset{?}{=} \tan^2\theta$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 \overset{?}{=} \tan^2\theta$$

$$\tan^2\theta = \tan^2\theta$$

8. $\frac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$

$$\frac{\cos^2\theta}{1 - \sin\theta} \overset{?}{=} 1 + \sin\theta$$

$$\frac{1 - \sin^2\theta}{1 - \sin\theta} \overset{?}{=} 1 + \sin\theta$$

$$\frac{(1 + \sin\theta)(1 - \sin\theta)}{1 - \sin\theta} \overset{?}{=} 1 + \sin\theta$$

$$1 + \sin\theta = 1 + \sin\theta$$

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14-4 Practice (Average)

Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \sec^2\theta$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} \stackrel{?}{=} \sec^2\theta$

$\frac{1}{\cos^2\theta} \stackrel{?}{=} \sec^2\theta$

$\sec^2\theta = \sec^2\theta$

2. $\frac{\cos^2\theta}{1 - \sin^2\theta} = 1$

$\frac{\cos^2\theta}{1 - \sin^2\theta} \stackrel{?}{=} 1$

$\frac{\cos^2\theta}{\cos^2\theta} \stackrel{?}{=} 1$

$1 = 1$

3. $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

$(1 + \sin\theta)(1 - \sin\theta) \stackrel{?}{=} \cos^2\theta$

$1 - \sin^2\theta \stackrel{?}{=} \cos^2\theta$

$\cos^2\theta = \cos^2\theta$

4. $\tan^4\theta + 2\tan^2\theta + 1 = \sec^4\theta$

$\tan^4\theta + 2\tan^2\theta + 1 \stackrel{?}{=} \sec^4\theta$

$(\tan^2\theta + 1)^2 \stackrel{?}{=} \sec^4\theta$

$(\sec^2\theta)^2 \stackrel{?}{=} \sec^4\theta$

$\sec^4\theta = \sec^4\theta$

5. $\cos^2\theta\cot^2\theta = \cot^2\theta - \cos^2\theta$

$\cos^2\theta\cot^2\theta \stackrel{?}{=} \cot^2\theta - \cos^2\theta$

$\cos^2\theta\cot^2\theta \stackrel{?}{=} \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$

$\cos^2\theta\cot^2\theta \stackrel{?}{=} \frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}$

$\cos^2\theta\cot^2\theta \stackrel{?}{=} \frac{(\cos^2\theta)(1 - \sin^2\theta)}{\sin^2\theta}$

$\cos^2\theta\cot^2\theta \stackrel{?}{=} \frac{\cos^2\theta}{1} \cdot \frac{\cos^2\theta}{\sin^2\theta}$

$\cos^2\theta\cot^2\theta = \cos^2\theta\cot^2\theta$

6. $(\sin^2\theta)(\csc^2\theta + \sec^2\theta) = \sec^2\theta$

$(\sin^2\theta)(\csc^2\theta + \sec^2\theta) \stackrel{?}{=} \sec^2\theta$

$(\sin^2\theta)\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right) \stackrel{?}{=} \sec^2\theta$

$1 + \frac{\sin^2\theta}{\cos^2\theta} \stackrel{?}{=} \sec^2\theta$

$1 + \tan^2\theta \stackrel{?}{=} \sec^2\theta$

$\sec^2\theta = \sec^2\theta$

7. PROJECTILES The square of the initial velocity of an object launched from the ground is $v^2 = \frac{2gh}{\sin^2\theta}$, where θ is the angle between the ground and the initial path, h is the maximum height reached, and g is the acceleration due to gravity. Verify the identity $\frac{2gh}{\sin^2\theta} = \frac{2gh\sec^2\theta}{\sec^2\theta - 1}$.

$\frac{2gh}{\sin^2\theta} = \frac{2gh}{1 - \cos^2\theta} = \frac{2gh}{1 - \frac{1}{\sec^2\theta}} = \frac{2gh}{\frac{\sec^2\theta - 1}{\sec^2\theta}} = \frac{2gh\sec^2\theta}{\sec^2\theta - 1}$

8. LIGHT The intensity of a light source measured in candles is given by $I = ER^2\sec\theta$, where E is the illuminance in foot candles on a surface, R is the distance in feet from the light source, and θ is the angle between the light beam and a line perpendicular to the surface. Verify the identity $ER^2(1 + \tan^2\theta)\cos\theta = ER^2\sec\theta$.

$ER^2(1 + \tan^2\theta)\cos\theta = ER^2\sec^2\theta\cos\theta = ER^2\sec^2\theta \cdot \frac{1}{\sec\theta} = ER^2\sec\theta$

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14-4 Word Problem Practice

Verifying Trigonometric Identities

GRAPHING FUNCTIONS For Exercises 1–3, use the following information.

Brandi is doing her trigonometry homework and needs to graph the function $y = \frac{\sin^2 x - \tan^2 x}{\sec^2 x}$. She thinks that it would be easier to graph the function if she could rewrite it in a simpler way, either without a denominator or as an expression containing only one trigonometric function. After some work, Brandi decides that she can graph $y = -\sin^4 x$ instead of the given function.

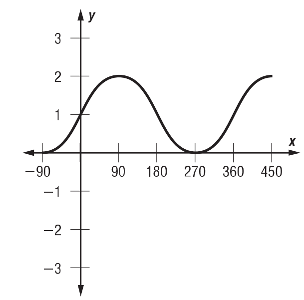
1. Is it possible for Brandi to simplify the function in the way she claims? **yes**

2. If Brandi graphs the given function and her simpler function on the same set of axes, what will she find? **She will see only one graph.**

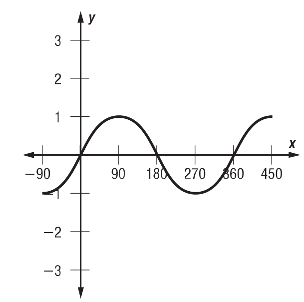
3. What does this mean about $\frac{\sin^2 x - \tan^2 x}{\sec^2 x}$ and $-\sin^4 x$?

The expressions are equal, meaning that $\frac{\sin^2 x - \tan^2 x}{\sec^2 x} = -\sin^4 x$ is an identity.

4. EXPERIMENTS Kyle is performing an experiment for his physics class. He uses a motion detector connected to a computer to collect data. The first time he performs the experiment, the computer produces the graph below. The graph is of the function $y = \frac{\cos^2 x}{1 - \sin x}$.



The second time he performs the experiment, the computer produces the graph below. The graph is of the $y = \sin x$.



Use the graphs to write an identity involving $\frac{\cos^2 x}{1 - \sin x}$ and $\sin x$.

$\frac{\cos^2 x}{1 - \sin x} = 1 + \sin x$

or $\frac{\cos^2 x}{1 - \sin x} - 1 = \sin x$

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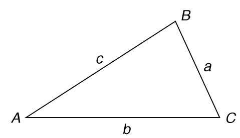
14-4 Enrichment

Heron's Formula

Heron's formula can be used to find the area of a triangle if you know the lengths of the three sides. Consider any triangle ABC. Let K represent the area of $\triangle ABC$. Then



$$K = \frac{1}{2}bc \sin A$$

$$K^2 = \frac{b^2c^2 \sin^2 A}{4} \quad \text{Square both sides.}$$

$$= \frac{b^2c^2(1 - \cos^2 A)}{4}$$

$$= \frac{b^2c^2(1 + \cos A)(1 - \cos A)}{4}$$

$$= \frac{b^2c^2}{4}\left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right)\left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right) \quad \text{Use the law of cosines.}$$

$$= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \cdot \frac{a + b - c}{2} \cdot \frac{a - b + c}{2} \quad \text{Simplify.}$$

Let $s = \frac{a + b + c}{2}$. Then $s - a = \frac{b + c - a}{2}$, $s - b = \frac{a + c - b}{2}$, $s - c = \frac{a + b - c}{2}$.

$$K^2 = s(s - a)(s - b)(s - c) \quad \text{Substitute.}$$

$$K = \sqrt{s(s - a)(s - b)(s - c)}$$

Heron's Formula	The area of $\triangle ABC$ is $\sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{a + b + c}{2}$.

Use Heron's formula to find the area of $\triangle ABC$.

1. $a = 3, b = 4.4, c = 7$
4.1

2. $a = 8.2, b = 10.3, c = 9.5$
36.8

3. $a = 31.3, b = 92.0, c = 67.9$
782.9

4. $a = 0.54, b = 1.32, c = 0.78$
no such triangle

5. $a = 321, b = 178, c = 298$
26,160.9

6. $a = 0.05, b = 0.08, c = 0.04$
0.00082

7. $a = 21.5, b = 33.0, c = 41.7$
351.6

8. $a = 2.08, b = 9.13, c = 8.99$
9.3

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14-5 Lesson Reading Guide

Sum and Difference of Angles Formulas

Pre-Activity **How are the sum and difference formulas used to describe communication interference?**

Read the introduction to Lesson 14-5 at the top of page 786 in your textbook.

Consider the functions $y = \sin x$ and $y = 2 \sin x$. Do the graphs of these two functions have *constructive* interference or *destructive* interference?
constructive

Reading the Lesson

1. Match each expression from the list on the left with an expression from the list on the right that is equal to it for all values of the variables. (Some of the expressions from the list on the right may be used more than once or not at all.)

a. $\sin(\alpha - \beta)$ **v**	**i.** $\sin \beta$
b. $\cos(\alpha + \beta)$ **vi**	**ii.** $\sin \alpha \cos \beta + \cos \alpha \sin \beta$
c. $\sin(180° + \beta)$ **vii**	**iii.** $-\cos \beta$
d. $\sin(180° - \beta)$ **i**	**iv.** $\cos \alpha \cos \beta + \sin \alpha \sin \beta$
e. $\cos(180° + \beta)$ **iii**	**v.** $\sin \alpha \cos \beta - \cos \alpha \sin \beta$
f. $\sin(\alpha + \beta)$ **ii**	**vi.** $\cos \alpha \cos \beta - \sin \alpha \sin \beta$
g. $\cos(90° - \beta)$ **i**	**vii.** $-\sin \beta$
h. $\cos(\alpha - \beta)$ **iv**	**viii.** $\cos \beta$

2. Which expressions are equal to sin 15°? (There may be more than one correct choice.)

A. $\sin 45° \cos 30° + \cos 45° \sin 30°$
B. $\sin 45° \cos 30° - \cos 45° \sin 30°$ **B and C**
C. $\sin 60° \cos 45° - \cos 60° \sin 45°$
D. $\cos 60° \cos 45° - \sin 60° \sin 45°$

Helping You Remember

3. Some students have trouble remembering which signs to use on the right-hand sides of the sum and difference of angle formulas. What is an easy way to remember this?

Sample answer: In the *sine* identities, the signs are the *same* on both sides. In the *cosine* identities, the signs are *opposite* on the two sides.

Lesson 14-5

Answers

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14-5 Study Guide and Intervention

Sum and Difference of Angles Formulas

Sum and Difference Formulas The following formulas are useful for evaluating an expression like sin 15° from the known values of sine and cosine of 60° and 45°.

Sum and Difference of Angles	The following identities hold true for all values of α and β. $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$ $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

Example **Find the exact value of each expression.**

a. cos 345°

$$\begin{aligned}\cos 345° &= \cos(300° + 45°)\\ &= \cos 300° \cdot \cos 45° - \sin 300° \cdot \sin 45°\\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

b. sin (−105°)

$$\begin{aligned}\sin(-105°) &= \sin(45° - 150°)\\ &= \sin 45° \cdot \cos 150° - \cos 45° \cdot \sin 150°\\ &= \frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= -\frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

Exercises

Find the exact value of each expression.

1. sin 105° $\frac{\sqrt{2}+\sqrt{6}}{4}$

2. cos 285° $\frac{\sqrt{6}-\sqrt{2}}{4}$

3. cos (−75°) $\frac{\sqrt{6}-\sqrt{2}}{4}$

4. cos (−165°) $-\frac{\sqrt{2}+\sqrt{6}}{4}$

5. sin 195° $\frac{\sqrt{2}-\sqrt{6}}{4}$

6. cos 420° $\frac{1}{2}$

7. sin (−75°) $-\frac{\sqrt{2}+\sqrt{6}}{4}$

8. cos 135° $-\frac{\sqrt{2}}{2}$

9. cos (−15°) $\frac{\sqrt{2}+\sqrt{6}}{4}$

10. sin 345° $\frac{\sqrt{2}-\sqrt{6}}{4}$

11. cos (−105°) $\frac{\sqrt{2}-\sqrt{6}}{4}$

12. sin 495° $\frac{\sqrt{2}}{2}$

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14-5 Study Guide and Intervention

Sum and Difference of Angles Formulas

Verify Identities You can also use the sum and difference of angles formulas to verify identities.

Example 1 **Verify that $\cos\left(\theta + \frac{3\pi}{2}\right) = \sin\theta$ is an identity.**

$\cos\left(\theta + \frac{3\pi}{2}\right) \stackrel{?}{=} \sin\theta$	Original equation
$\cos\theta \cdot \cos\frac{3\pi}{2} - \sin\theta \cdot \sin\frac{3\pi}{2} \stackrel{?}{=} \sin\theta$	Sum of Angles Formula
$\cos\theta \cdot 0 - \sin\theta \cdot (-1) \stackrel{?}{=} \sin\theta$	Evaluate each expression.
$\sin\theta = \sin\theta$	Simplify.

Example 2 **Verify that $\sin\left(\theta - \frac{\pi}{2}\right) + \cos(\theta + \pi) = -2\cos\theta$ is an identity.**

$\sin\left(\theta - \frac{\pi}{2}\right) + \cos(\theta + \pi) \stackrel{?}{=} -2\cos\theta$	Original equation
$\sin\theta \cdot \cos\frac{\pi}{2} - \cos\theta \cdot \sin\frac{\pi}{2} + \cos\theta \cdot \cos\pi - \sin\theta \cdot \sin\pi \stackrel{?}{=} -2\cos\theta$	Sum and Difference of Angles Formulas
$\sin\theta \cdot 0 - \cos\theta \cdot 1 + \cos\theta \cdot (-1) - \sin\theta \cdot 0 \stackrel{?}{=} -2\cos\theta$	Evaluate each expression.
$-2\cos\theta = -2\cos\theta$	Simplify.

Exercises

Verify that each of the following is an identity.

1. $\sin(90° + \theta) = \cos\theta$

$$\begin{aligned}\sin 90° \cdot \cos\theta + \cos 90° \cdot \sin\theta &\stackrel{?}{=} \cos\theta\\ 1 \cdot \cos\theta + 0 \cdot \sin\theta &\stackrel{?}{=} \cos\theta\\ \cos\theta &= \cos\theta\end{aligned}$$

2. $\cos(270° + \theta) = \sin\theta$

$$\begin{aligned}\cos 270° \cdot \cos\theta - \sin 270° \cdot \sin\theta &\stackrel{?}{=} \sin\theta\\ 0 \cdot \cos\theta - (-1) \cdot \sin\theta &\stackrel{?}{=} \sin\theta\\ \sin\theta &= \sin\theta\end{aligned}$$

3. $\sin\left(\frac{2\pi}{3} - \theta\right) + \cos\left(\theta - \frac{5\pi}{6}\right) = \sin\theta$

$$\begin{aligned}\sin\frac{2\pi}{3} \cdot \cos\theta - \cos\frac{2\pi}{3} \cdot \sin\theta + \cos\theta \cdot \cos\frac{5\pi}{6} + \sin\theta \cdot \sin\frac{5\pi}{6} &\stackrel{?}{=} \sin\theta\\ \frac{\sqrt{3}}{2} \cdot \cos\theta - \left(-\frac{1}{2}\right) \cdot \sin\theta + \cos\theta\left(-\frac{\sqrt{3}}{2}\right) + \sin\theta \cdot \frac{1}{2} &\stackrel{?}{=} \sin\theta\\ \sin\theta &= \sin\theta\end{aligned}$$

4. $\cos\left(\frac{3\pi}{4} + \theta\right) - \sin\left(\theta - \frac{\pi}{4}\right) = -\sqrt{2}\sin\theta$

$$\begin{aligned}\cos\frac{3\pi}{4} \cdot \cos\theta - \sin\frac{3\pi}{4} \cdot \sin\theta - \left(\sin\theta \cdot \cos\frac{\pi}{4} - \cos\theta \cdot \sin\frac{\pi}{4}\right) &\stackrel{?}{=} -\sqrt{2}\sin\theta\\ \left(-\frac{\sqrt{2}}{2}\right) \cdot \cos\theta - \frac{\sqrt{2}}{2} \cdot \sin\theta - \left(\sin\theta \cdot \frac{\sqrt{2}}{2} - \cos\theta \cdot \frac{\sqrt{2}}{2}\right) &\stackrel{?}{=} -\sqrt{2}\sin\theta\\ -\sqrt{2}\sin\theta &= -\sqrt{2}\sin\theta\end{aligned}$$

Lesson 14-5

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14-5 Skills Practice

Sum and Difference of Angles Formulas

Find the exact value of each expression.

1. sin 330° $-\frac{1}{2}$

2. cos (−165°) $\frac{-\sqrt{6}-\sqrt{2}}{4}$

3. sin (−225°) $\frac{\sqrt{2}}{2}$

4. cos 135° $-\frac{\sqrt{2}}{2}$

5. sin (−45°) $-\frac{\sqrt{2}}{2}$

6. cos 210° $-\frac{\sqrt{3}}{2}$

7. cos (−135°) $-\frac{\sqrt{2}}{2}$

8. sin 75° $\frac{\sqrt{6}+\sqrt{2}}{4}$

9. sin (−195°) $\frac{\sqrt{6}-\sqrt{2}}{4}$

Verify that each of the following is an identity.

10. $\sin(90° + \theta) = \cos\theta$

$\sin(90° + \theta) \stackrel{?}{=} \cos\theta$
$\sin 90° \cos\theta + \cos 90° \sin\theta \stackrel{?}{=} \cos\theta$
$1 \cos\theta + 0 \sin\theta \stackrel{?}{=} \cos\theta$
$\cos\theta = \cos\theta$

11. $\sin(180° + \theta) = -\sin\theta$

$\sin(180° + \theta) \stackrel{?}{=} -\sin\theta$
$\sin 180° \cos\theta + \cos 180° \sin\theta \stackrel{?}{=} -\sin\theta$
$0 \cos\theta + (-1) \sin\theta \stackrel{?}{=} -\sin\theta$
$-\sin\theta = -\sin\theta$

12. $\cos(270° - \theta) = -\sin\theta$

$\cos(270° - \theta) \stackrel{?}{=} -\sin\theta$
$\cos 270° \cos\theta + \sin 270° \sin\theta \stackrel{?}{=} -\sin\theta$
$0 \cos\theta + (-1) \sin\theta \stackrel{?}{=} -\sin\theta$
$-\sin\theta = -\sin\theta$

13. $\cos(\theta - 90°) = \sin\theta$

$\cos(\theta - 90°) \stackrel{?}{=} \sin\theta$
$\cos\theta \cos 90° + \sin\theta \sin 90° \stackrel{?}{=} \sin\theta$
$(\cos\theta)(0) + (\sin\theta)(1) \stackrel{?}{=} \sin\theta$
$\sin\theta = \sin\theta$

14. $\sin\left(\theta - \frac{\pi}{2}\right) = -\cos\theta$

$\sin\left(\theta - \frac{\pi}{2}\right) \stackrel{?}{=} -\cos\theta$
$\sin\theta \cos\frac{\pi}{2} - \cos\theta \sin\frac{\pi}{2} \stackrel{?}{=} -\cos\theta$
$(\sin\theta)(0) - (\cos\theta)(1) \stackrel{?}{=} -\cos\theta$
$-\cos\theta = -\cos\theta$

15. $\cos(\pi + \theta) = -\cos\theta$

$\cos(\pi + \theta) \stackrel{?}{=} -\cos\theta$
$\cos\pi \cos\theta - \sin\pi \sin\theta \stackrel{?}{=} -\cos\theta$
$-1 \cos\theta - 0 \sin\theta \stackrel{?}{=} -\cos\theta$
$-\cos\theta = -\cos\theta$

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14-5 Practice (Average)

Sum and Difference of Angles Formulas

Find the exact value of each expression.

1. cos 75° $\frac{\sqrt{6}-\sqrt{2}}{4}$

2. cos 375° $\frac{\sqrt{6}+\sqrt{2}}{4}$

3. sin (−165°) $\frac{\sqrt{2}-\sqrt{6}}{4}$

4. sin (−105°) $\frac{-\sqrt{2}-\sqrt{6}}{4}$

5. sin 150° $\frac{1}{2}$

6. cos 240° $-\frac{1}{2}$

7. sin 225° $-\frac{\sqrt{2}}{2}$

8. sin (−75°) $\frac{-\sqrt{2}-\sqrt{6}}{4}$

9. sin 195° $\frac{\sqrt{2}-\sqrt{6}}{4}$

Verify that each of the following is an identity.

10. $\cos(180° - \theta) = -\cos\theta$

$\cos(180° - \theta) \stackrel{?}{=} -\cos\theta$
$\cos 180° \cos\theta + \sin 180° \sin\theta \stackrel{?}{=} -\cos\theta$
$-1 \cos\theta + 0 \sin\theta \stackrel{?}{=} -\cos\theta$
$-\cos\theta = -\cos\theta$

11. $\sin(360° + \theta) = \sin\theta$

$\sin(360° + \theta) \stackrel{?}{=} \sin\theta$
$\sin 360° \cos\theta + \cos 360° \sin\theta \stackrel{?}{=} \sin\theta$
$0 \cos\theta + 1 \sin\theta \stackrel{?}{=} \sin\theta$
$\sin\theta = \sin\theta$

12. $\sin(45° + \theta) - \sin(45° - \theta) = \sqrt{2}\sin\theta$

$\sin(45° + \theta) - \sin(45° - \theta)$
$\stackrel{?}{=} \sin 45° \cos\theta + \cos 45° \sin\theta - (\sin 45° \cos\theta - \cos 45° \sin\theta)$
$\stackrel{?}{=} 2 \cdot \cos 45° \sin\theta$
$\stackrel{?}{=} 2 \cdot \frac{\sqrt{2}}{2} \cdot \sin\theta$
$= \sqrt{2}\sin\theta$

13. $\cos\left(x - \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{3}\right) = \sin x$

$\cos\left(x - \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{3}\right)$
$\stackrel{?}{=} \cos x \cos\frac{\pi}{6} + \sin x \sin\frac{\pi}{6} + \sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}$
$\stackrel{?}{=} \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x + \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$
$= \sin x$

14. SOLAR ENERGY On March 21, the maximum amount of solar energy that falls on a square foot of ground at a certain location is given by $E \sin(90° - \phi)$, where ϕ is the latitude of the location and E is a constant. Use the difference of angles formula to find the amount of solar energy, in terms of $\cos\phi$, for a location that has a latitude of ϕ. $E \cos\phi$

ELECTRICITY In Exercises 15 and 16, use the following information.

In a certain circuit carrying alternating current, the formula $i = 2\sin(120t)$ can be used to find the current i in amperes after t seconds.

15. Rewrite the formula using the sum of two angles. Sample answer: $i = 2\sin(90t + 30t)$

16. Use the sum of angles formula to find the exact current at $t = 1$ second. $\sqrt{3}$ amperes

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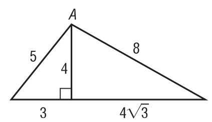
14-5 Word Problem Practice

Sum and Difference of Angles Formulas

ART For Exercises 1–4, use the following information.

As part of a mosaic that an artist is making, she places two right triangular tiles together to make a new triangular piece. One tile has lengths of 3 inches, 4 inches, and 5 inches. The other tile has lengths 4 inches, $4\sqrt{3}$ inches, and 8 inches. The pieces are placed with the sides of 4 inches against each other, as shown in the figure below.



1. What is the exact value of the sine of angle A? $\frac{3 + 4\sqrt{3}}{10}$

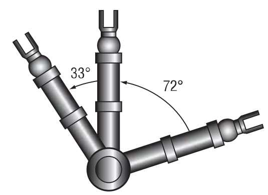
2. What is the exact value of the cosine of angle A? $\frac{4 - 3\sqrt{3}}{10}$

3. What is the measure of angle A? **96.9°**

4. Is the new triangle formed from the two triangles also a right triangle? **no**

MANUFACTURING For Exercises 5–7, use the following information.

A robotic arm performs two operations in the course of manufacturing each car door that comes down the assembly line. As the door arrives in front of the robotic arm, the arm is parallel to the assembly line. Once the door is in place, the arm rotates counterclockwise 72° to perform the first operation. After performing the first operation, the arm rotates another 33° counterclockwise to perform the second operation. After the second operation is completed, the robotic arm rotates clockwise back to its starting position.



5. After completing the second operation, through what angle does the robotic arm rotate to return to its starting position? **105°**

6. What is the exact value of the sine of the angle through which the arm rotates to return to its starting position? $\frac{\sqrt{6} + \sqrt{2}}{4}$

7. What is the exact value of the cosine of the angle through which the arm rotates to return to its starting position? $\frac{\sqrt{2} - \sqrt{6}}{4}$

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14-5 Enrichment

Identities for the Products of Sines and Cosines

By adding the identities for the sines of the sum and difference of the measures of two angles, a new identity is obtained.

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$(i)\quad \sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$$

This new identity is useful for expressing certain products as sums.

Example **Write $\sin 3\theta \cos\theta$ as a sum.**

In the identity let $\alpha = 3\theta$ and $\beta = \theta$ so that
$2\sin 3\theta\cos\theta = \sin(3\theta + \theta) + \sin(3\theta - \theta)$. Thus,
$\sin 3\theta\cos\theta = \frac{1}{2}\sin 4\theta + \frac{1}{2}\sin 2\theta$.

By subtracting the identities for $\sin(\alpha + \beta)$ and $\sin(\alpha - \beta)$, a similar identity for expressing a product as a difference is obtained.

$$(ii)\quad \sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\alpha\sin\beta$$

Solve.

1. Use the identities for $\cos(\alpha + \beta)$ and $\cos(\alpha - \beta)$ to find identities for expressing the products $2\cos\alpha\cos\beta$ and $2\sin\alpha\sin\beta$ as a sum or difference.
   $2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
   $2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

2. Find the value of $\sin 105°\cos 75°$ without using tables.
   $\frac{1}{2}[\sin(105° + 75°) + \sin(105° - 75°)]$;
   $\frac{1}{2}\left(0 + \frac{1}{2}\right); \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

3. Express $\cos\theta\sin\frac{\theta}{2}$ as a difference.
   $2\cos\theta\sin\frac{\theta}{2} = \sin\left(\theta + \frac{\theta}{2}\right) - \sin\left(\theta - \frac{\theta}{2}\right)$
   $\cos\theta\sin\frac{\theta}{2} = \frac{1}{2}\sin\frac{3\theta}{2} - \frac{1}{2}\sin\frac{\theta}{2}$

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14-6 Lesson Reading Guide

Double-Angle and Half-Angle Formulas

Pre-Activity **How can trigonometric functions be used to describe music?**

Read the introduction to Lesson 14-6 at the top of page 791 in your textbook.

Suppose that the equation for the second harmonic is $y = \sin a\theta$. Then what would be the equations for the fundamental tone (first harmonic), third harmonic, fourth harmonic, and fifth harmonic?

$y = \sin 0.5a\theta$; $y = \sin 1.5a\theta$; $y = \sin 2a\theta$; $y = \sin 2.5a\theta$

Reading the Lesson

1. Match each expression from the list on the left with *all* expressions from the list on the right that are equal to it for all values of β.

a. $\sin \frac{\beta}{2}$ **v** — **i.** $2 \sin \beta \cos \beta$

b. $\cos 2\beta$ **ii and iii** — **ii.** $1 - 2\sin^2 \beta$

c. $\cos \frac{\beta}{2}$ **iv** — **iii.** $\cos^2 \beta - \sin^2 \beta$

d. $\sin 2\beta$ **i** — **iv.** $\pm\sqrt{\frac{1 + \cos \beta}{2}}$

v. $\pm\sqrt{\frac{1 - \cos \beta}{2}}$

2. Determine whether you would use the *positive* or *negative* square root in the half-angle identities for $\sin \frac{\alpha}{2}$ and $\cos \frac{\alpha}{2}$ in each of the following situations. (Do not actually calculate $\sin \frac{\alpha}{2}$ and $\cos \frac{\alpha}{2}$.)

a. $\sin \frac{\alpha}{2}$, if $\cos \alpha = \frac{2}{5}$ and α is in Quadrant I **positive**

b. $\cos \frac{\alpha}{2}$, if $\cos \alpha = -0.9$ and α is in Quadrant II **positive**

c. $\cos \frac{\alpha}{2}$, if $\sin \alpha = -0.75$ and α is in Quadrant III **negative**

d. $\sin \frac{\alpha}{2}$, if $\sin \alpha = -0.8$ and α is in Quadrant IV **positive**

Helping You Remember

3. Many students find it difficult to remember a large number of identities. How can you obtain all three of the identities for $\cos 2\theta$ by remembering only one of them and using a Pythagorean identity?

Sample answer: Just remember the identity $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$. Using the Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$, you can substitute either $1 - \sin^2 \theta$ for $\cos^2 \theta$ or $1 - \cos^2 \theta$ for $\sin^2 \theta$ to get the other two identities for $\cos 2\theta$.

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14-6 Study Guide and Intervention

Double-Angle and Half-Angle Formulas

Double-Angle Formulas

Double-Angle Formulas	The following identities hold true for all values of θ. $\sin 2\theta = 2 \sin \theta \cdot \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\cos 2\theta = 1 - 2\sin^2 \theta$ $\cos 2\theta = 2\cos^2 \theta - 1$

Example **Find the exact values of $\sin 2\theta$ and $\cos 2\theta$ if $\sin \theta = -\frac{9}{10}$ and $180° < \theta < 270°$.**

First, find the value of $\cos \theta$.

$\cos^2 \theta = 1 - \sin^2 \theta$ $\cos^2 \theta + \sin^2 \theta = 1$

$\cos^2 \theta = 1 - \left(-\frac{9}{10}\right)^2$ $\sin \theta = -\frac{9}{10}$

$\cos^2 \theta = \frac{19}{100}$

$\cos \theta = \pm\frac{\sqrt{19}}{10}$

Since θ is in the third quadrant, $\cos \theta$ is negative. Thus $\cos \theta = -\frac{\sqrt{19}}{10}$.

To find $\sin 2\theta$, use the identity $\sin 2\theta = 2 \sin \theta \cdot \cos \theta$.

$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$

$= 2\left(-\frac{9}{10}\right)\left(-\frac{\sqrt{19}}{10}\right)$

$= \frac{9\sqrt{19}}{50}$

The value of $\sin 2\theta$ is $\frac{9\sqrt{19}}{50}$.

To find $\cos 2\theta$, use the identity $\cos 2\theta = 1 - 2\sin^2 \theta$.

$\cos 2\theta = 1 - 2\sin^2 \theta$

$= 1 - 2\left(-\frac{9}{10}\right)^2$

$= -\frac{31}{50}$.

The value of $\cos 2\theta$ is $-\frac{31}{50}$.

Exercises

Find the exact values of $\sin 2\theta$ and $\cos 2\theta$ for each of the following.

1. $\sin \theta = \frac{1}{4}$, $0° < \theta < 90°$ **$\frac{\sqrt{15}}{8}, \frac{7}{8}$**

2. $\sin \theta = -\frac{1}{8}$, $270° < \theta < 360°$ **$-\frac{3\sqrt{7}}{32}, \frac{31}{32}$**

3. $\cos \theta = -\frac{3}{5}$, $180° < \theta < 270°$ **$\frac{24}{25}, -\frac{7}{25}$**

4. $\cos \theta = -\frac{4}{5}$, $90° < \theta < 180°$ **$-\frac{24}{25}, \frac{7}{25}$**

5. $\sin \theta = -\frac{3}{5}$, $270° < \theta < 360°$ **$-\frac{24}{25}, \frac{7}{25}$**

6. $\cos \theta = -\frac{2}{3}$, $90° < \theta < 180°$ **$-\frac{4\sqrt{5}}{9}, -\frac{1}{9}$**

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14-6 Study Guide and Intervention *(continued)*

Double-Angle and Half-Angle Formulas

Half-Angle Formulas

Half-Angle Formulas	The following identities hold true for all values of α. $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

Example **Find the exact value of $\sin\frac{\alpha}{2}$ if $\sin\alpha = \frac{2}{3}$ and $90° < \alpha < 180°$.**

First find $\cos\alpha$.

$\cos^2\alpha = 1 - \sin^2\alpha$ — $\cos^2\alpha + \sin^2\alpha = 1$

$\cos^2\alpha = 1 - \left(\frac{2}{3}\right)^2$ — $\sin\alpha = \frac{2}{3}$

$\cos^2\alpha = \frac{5}{9}$ — Simplify.

$\cos\alpha = \pm\frac{\sqrt{5}}{3}$ — Take the square root of each side.

Since α is in the second quadrant, $\cos\alpha = -\frac{\sqrt{5}}{3}$.

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ — Half-Angle formula

$= \pm\sqrt{\frac{1-\left(-\frac{\sqrt{5}}{3}\right)}{2}}$ — $\cos\alpha = -\frac{\sqrt{5}}{3}$

$= \pm\sqrt{\frac{3+\sqrt{5}}{6}}$ — Simplify.

$= \pm\frac{\sqrt{18+6\sqrt{5}}}{6}$ — Rationalize.

Since α is between 90° and 180°, $\frac{\alpha}{2}$ is between 45° and 90°. Thus $\sin\frac{\alpha}{2}$ is positive and equals $\frac{\sqrt{18+6\sqrt{5}}}{6}$.

Exercises

Find the exact value of $\sin\frac{\alpha}{2}$ and $\cos\frac{\alpha}{2}$ for each of the following.

1. $\cos\alpha = -\frac{3}{5}$, $180° < \alpha < 270°$
$\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}$

2. $\cos\alpha = -\frac{4}{5}$, $90° < \alpha < 180°$
$\frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10}$

3. $\sin\alpha = -\frac{3}{5}$, $270° < \alpha < 360°$
$\frac{\sqrt{10}}{10}, -\frac{3\sqrt{10}}{10}$

4. $\cos\alpha = -\frac{2}{3}$, $90° < \alpha < 180°$
$\frac{\sqrt{30}}{6}, \frac{\sqrt{6}}{6}$

Find the exact value of each expression by using the half-angle formulas.

5. $\cos 22\frac{1}{2}°$ $\frac{\sqrt{2+\sqrt{2}}}{2}$

6. $\sin 67.5°$ $\frac{\sqrt{2+\sqrt{2}}}{2}$

7. $\cos\frac{7\pi}{8}$ $-\frac{\sqrt{2+\sqrt{2}}}{2}$

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14-6 Skills Practice

Double-Angle and Half-Angle Formulas

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$ for each of the following.

1. $\cos\theta = \frac{7}{25}$, $0° < \theta < 90°$
$\frac{336}{625}, -\frac{527}{625}, \frac{3}{5}, \frac{4}{5}$

2. $\sin\theta = -\frac{4}{5}$, $180° < \theta < 270°$
$-\frac{24}{25}, -\frac{7}{25}, \frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}$

3. $\sin\theta = \frac{40}{41}$, $90° < \theta < 180°$
$-\frac{720}{1681}, -\frac{1519}{1681}, \frac{5\sqrt{41}}{41}, \frac{4\sqrt{41}}{41}$

4. $\cos\theta = \frac{3}{7}$, $270° < \theta < 360°$
$-\frac{12\sqrt{10}}{49}, -\frac{31}{49}, \frac{\sqrt{14}}{7}, -\frac{\sqrt{35}}{7}$

5. $\cos\theta = -\frac{3}{5}$, $90° < \theta < 180°$
$-\frac{24}{25}, -\frac{7}{25}, \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}$

6. $\sin\theta = \frac{5}{13}$, $0° < \theta < 90°$
$\frac{120}{169}, \frac{119}{169}, \frac{\sqrt{26}}{26}, \frac{5\sqrt{26}}{26}$

Find the exact value of each expression by using the half-angle formulas.

7. $\cos 22\frac{1}{2}°$ $\frac{\sqrt{2+\sqrt{2}}}{2}$

8. $\sin 165°$ $\frac{\sqrt{2-\sqrt{3}}}{2}$

9. $\cos 105°$ $-\frac{\sqrt{2-\sqrt{3}}}{2}$

10. $\sin\frac{\pi}{8}$ $\frac{\sqrt{2-\sqrt{2}}}{2}$

11. $\sin\frac{15\pi}{8}$ $-\frac{\sqrt{2-\sqrt{2}}}{2}$

12. $\cos 75°$ $\frac{\sqrt{2-\sqrt{3}}}{2}$

Verify that each of the following is an identity.

13. $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\sin 2\theta \stackrel{?}{=} \frac{2\tan\theta}{1+\tan^2\theta}$

$2\sin\theta\cos\theta \stackrel{?}{=} \frac{2\tan\theta}{\sec^2\theta}$

$2\sin\theta\cos\theta \stackrel{?}{=} 2\frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta$

$2\sin\theta\cos\theta = 2\sin\theta\cos\theta$

14. $\tan\theta + \cot\theta = 2\csc 2\theta$

$\tan\theta + \cot\theta \stackrel{?}{=} 2\csc 2\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \frac{2}{\sin 2\theta}$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \frac{2}{2\sin\theta\cos\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} \stackrel{?}{=} \frac{1}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

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14-6 Practice (Average)

Double-Angle and Half-Angle Formulas

Find the exact values of sin 2θ, cos 2θ, sin $\frac{\theta}{2}$, and cos $\frac{\theta}{2}$ for each of the following.

1. $\cos\theta = \frac{5}{13}$, $0° < \theta < 90°$

   $\frac{120}{169}, -\frac{119}{169}, \frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13}$

2. $\sin\theta = \frac{8}{17}$, $90° < \theta < 180°$

   $-\frac{240}{289}, \frac{161}{289}, \frac{4\sqrt{17}}{17}, \frac{\sqrt{17}}{17}$

3. $\cos\theta = \frac{1}{4}$, $270° < \theta < 360°$

   $-\frac{\sqrt{15}}{8}, -\frac{7}{8}, \frac{\sqrt{6}}{4}, \frac{\sqrt{10}}{4}$

4. $\sin\theta = -\frac{2}{3}$, $180° < \theta < 270°$

   $\frac{4\sqrt{5}}{9}, \frac{1}{9}, \frac{\sqrt{18+6\sqrt{5}}}{6}, -\frac{\sqrt{18-6\sqrt{5}}}{6}$

Find the exact value of each expression by using the half-angle formulas.

5. tan 105° $\quad -2-\sqrt{3}$

6. tan 15° $\quad 2-\sqrt{3}$

7. cos 67.5° $\quad \frac{\sqrt{2-\sqrt{2}}}{2}$

8. $\sin\left(-\frac{\pi}{8}\right)$ $\quad -\frac{\sqrt{2-\sqrt{2}}}{2}$

Verify that each of the following is an identity.

9. $\sin^2\frac{\theta}{2} = \frac{\tan\theta - \sin\theta}{2\tan\theta}$

$$\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2 \stackrel{?}{=} \frac{\tan\theta-\sin\theta}{2\tan\theta};$$
$$\frac{1-\cos\theta}{2} \stackrel{?}{=} \frac{\frac{\tan\theta}{\tan\theta}-\frac{\sin\theta}{\tan\theta}}{2\,\frac{\tan\theta}{\tan\theta}};\ \frac{1-\cos\theta}{2} = \frac{1-\cos\theta}{2}$$

10. $\sin 4\theta = 4\cos 2\theta \sin\theta\cos\theta$

$$\sin 4\theta \stackrel{?}{=} 4\cos 2\theta\sin\theta\cos\theta$$
$$\sin 2(2\theta) \stackrel{?}{=} 4\cos 2\theta\sin\theta\cos\theta$$
$$2\sin 2\theta\cos 2\theta \stackrel{?}{=} 4\cos 2\theta\sin\theta\cos\theta$$
$$2(2\sin\theta\cos\theta)(\cos 2\theta) \stackrel{?}{=} 4\cos 2\theta\sin\theta\cos\theta$$
$$4\cos 2\theta\sin\theta\cos\theta = 4\cos 2\theta\sin\theta\cos\theta$$

11. **AERIAL PHOTOGRAPHY** In aerial photography, there is a reduction in film exposure for any point X not directly below the camera. The reduction E_θ is given by $E_\theta = E_0\cos^4\theta$, where θ is the angle between the perpendicular line from the camera to the ground and the line from the camera to point X, and E_0 is the exposure for the point directly below the camera. Using the identity $2\sin^2\theta = 1 - \cos 2\theta$, verify that $E_0\cos^4\theta = E_0\left(\frac{1}{2}+\frac{\cos 2\theta}{2}\right)^2$.

$$E_0\cos^4\theta = E_0(\cos^2\theta)^2 = E_0(1-\sin^2\theta)^2 = E_0\left(1-\frac{2\sin^2\theta}{2}\right)^2 =$$
$$E_0\left(1-\frac{1-\cos 2\theta}{2}\right)^2 = E_0\left(\frac{1}{2}+\frac{\cos 2\theta}{2}\right)^2$$

12. **IMAGING** A scanner takes thermal images from altitudes of 300 to 12,000 meters. The width W of the swath covered by the image is given by $W = 2H'\tan\theta$, where H' is the height and θ is half the scanner's field of view. Verify that $\frac{2H'\sin 2\theta}{1+\cos 2\theta} = 2H'\tan\theta$.

$$\frac{2H'\sin 2\theta}{1+\cos 2\theta} = \frac{4H'\sin\theta\cos\theta}{1+(2\cos^2\theta-1)} = \frac{4H'\sin\theta\cos\theta}{2\cos^2\theta} = \frac{2H'\sin\theta}{\cos\theta} = 2H'\tan\theta$$

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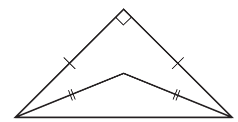
14-6 Word Problem Practice

Double-Angle and Half-Angle Formulas

GEOMETRY For Exercises 1–4, use the following information.

The large triangle shown in the figure below is an isosceles right triangle. The small triangle inside the large triangle was formed by bisecting each of the isosceles angles of the right triangle.



1. What is the exact value of the sine of either of the congruent angles of the small triangle?

   $\frac{\sqrt{2-\sqrt{2}}}{2}$

2. What is the exact value of the cosine of either of the congruent angles of the small triangle?

   $\frac{\sqrt{2+\sqrt{2}}}{2}$

3. What is the exact value of the sine of the obtuse angle of the small triangle?

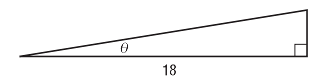
   $\frac{\sqrt{2}}{2}$

4. What is the exact value of the cosine of the obtuse angle of the small triangle?

   $-\frac{\sqrt{2}}{2}$

RAMPS For Exercises 5 and 6, use the following information.

A ramp for loading goods onto a truck was mistakenly built with the dimensions shown in the figure below. The degree measure of the angle the ramp makes with the ground should have been twice the degree measure of the angle shown in the figure.



5. Find the exact values of the sine and cosine of the angle the ramp should have made with the ground.

   $\frac{9}{41}, \frac{40}{41}$

6. If the ramp had been built properly, what would the degree measures of the two acute angles have been?

   12.7°, 77.3°

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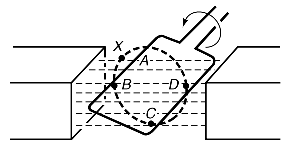
14-6 Enrichment

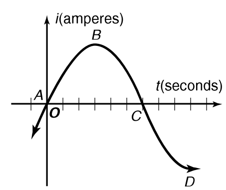
Alternating Current

The figure at the right represents an alternating current generator. A rectangular coil of wire is suspended between the poles of a magnet. As the coil of wire is rotated, it passes through the magnetic field and generates current.



As point X on the coil passes through the points A and C, its motion is along the direction of the magnetic field between the poles. Therefore, no current is generated. However, through points B and D, the motion of X is perpendicular to the magnetic field. This induces maximum current in the coil. Between A and B, B and C, C and D, and D and A, the current in the coil will have an intermediate value. Thus, the graph of the current of an alternating current generator is closely related to the sine curve.

The maximum current may have a positive or negative value.



The actual current, i, in a household current is given by $i = I_M \sin(120\pi t + \alpha)$ where I_M is the maximum value of the current, t is the elapsed time in seconds, and α is the angle determined by the position of the coil at time t_n.

Example **If $\alpha = \frac{\pi}{2}$, find a value of t for which $i = 0$.**

If $i = 0$, then $I_M \sin(120\pi t + \alpha) = 0$. $\quad i = I_M \sin(120\pi t + \alpha)$

Since $I_M \neq 0$, $\sin(120\pi t + \alpha) = 0$. $\quad$ If $ab = 0$ and $a \neq 0$, then $b = 0$.

Let $120\pi t + \alpha = s$. Thus, $\sin s = 0$.

$s = \pi$ because $\sin \pi = 0$.

$120\pi t + \alpha = \pi$ $\quad$ Substitute $120\pi t + \alpha$ for s.

$120\pi t + \frac{\pi}{2} = \pi$ $\quad$ Substitute $\frac{\pi}{2}$ for α.

$= \frac{1}{240}$ $\quad$ Solve for t.

This solution is the first positive value of t that satisfies the problem.

Using the equation for the actual current in a household circuit, $i = I_M \sin(120\pi t + \alpha)$, solve each problem. For each problem, find the first positive value of t.

1. If $\alpha = 0$, find a value of t for which $i = 0$. $t = \frac{1}{120}$

2. If $\alpha = 0$, find a value of t for which $i = +I_M$. $t = \frac{1}{240}$

3. If $\alpha = \frac{\pi}{2}$, find a value of t for which $i = -I_M$. $t = \frac{1}{120}$

4. If $\alpha = \frac{\pi}{4}$, find a value of t for which $i = 0$. $t = \frac{1}{160}$

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14-7 Study Guide and Intervention

Solving Trigonometric Equations

Solve Trigonometric Equations You can use trigonometric identities to solve trigonometric equations, which are true for only certain values of the variable.

Example 1 **Find all solutions of $4 \sin^2 \theta - 1 = 0$ for the interval $0° < \theta < 360°$.**

$4 \sin^2 \theta - 1 = 0$

$4 \sin^2 \theta = 1$

$\sin^2 \theta = \frac{1}{4}$

$\sin \theta = \pm\frac{1}{2}$

$\theta = 30°, 150°, 210°, 330°$

Example 2 **Solve $\sin 2\theta + \cos \theta = 0$ for all values of θ. Give your answer in both radians and degrees.**

$\sin 2\theta + \cos \theta = 0$

$2 \sin \theta \cos \theta + \cos \theta = 0$

$\cos \theta (2 \sin \theta + 1) = 0$

$\cos \theta = 0$ or $2 \sin \theta + 1 = 0$

$\sin \theta = -\frac{1}{2}$

$\theta = 90° + k \cdot 180°$; $\quad \theta = 210° + k \cdot 360°$,

$\theta = \frac{\pi}{2} + k \cdot \pi$ $\quad 330° + k \cdot 360°$;

$\theta = \frac{7\pi}{6} + k \cdot 2\pi$,

$\frac{11\pi}{6} + k \cdot 2\pi$

Exercises

Find all solutions of each equation for the given interval.

1. $2 \cos^2 \theta + \cos \theta = 1, 0 \leq \theta < 2\pi$
   $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

2. $\sin^2 \theta \cos^2 \theta = 0, 0 \leq \theta < 2\pi$
   $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

3. $\cos 2\theta = \frac{\sqrt{3}}{2}, 0° \leq \theta < 360°$
   $15°, 165°, 195°, 345°$

4. $2 \sin \theta - \sqrt{3} = 0, 0 \leq \theta < 2\pi$
   $\frac{\pi}{3}, \frac{2\pi}{3}$

Solve each equation for all values of θ if θ is measured in radians.

5. $4 \sin^2 \theta - 3 = 0$
   $\frac{\pi}{3} + k \cdot \pi, \frac{2\pi}{3} + k \cdot \pi$

6. $2 \cos \theta \sin \theta + \cos \theta = 0$
   $\frac{\pi}{2} + k \cdot 2\pi, \frac{3\pi}{2} + k \cdot 2\pi, \frac{7\pi}{6} + k \cdot 2\pi, \frac{11\pi}{6} + k \cdot 2\pi$

Solve each equation for all values of θ if θ is measured in degrees.

7. $\cos 2\theta + \sin^2 \theta = \frac{1}{2}$
   $45° + k \cdot 90°$

8. $\tan 2\theta = -1$
   $67.5° + k \cdot 360°, 157.5° + k \cdot 360°$

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14-7 Study Guide and Intervention

Solving Trigonometric Equations

Use Trigonometric Equations

Example LIGHT **Snell's law says that $\sin \alpha = 1.33 \sin \beta$, where α is the angle at which a beam of light enters water and β is the angle at which the beam travels through the water. If a beam of light enters water at 42°, at what angle does the light travel through the water?**

$\sin \alpha = 1.33 \sin \beta$	Original equation
$\sin 42° = 1.33 \sin \beta$	$\alpha = 42°$
$\sin \beta = \frac{\sin 42°}{1.33}$	Divide each side by 1.33.
$\sin \beta \approx 0.5031$	Use a calculator.
$\beta \approx 30.2°$	Take the arcsin of each side.

The light travels through the water at an angle of approximately 30.2°.

Exercises

1. A 6-foot pipe is propped on a 3-foot tall packing crate that sits on level ground. One foot of the pipe extends above the top of the crate and the other end rests on the ground. What angle does the pipe form with the ground? **36.9°**

2. At 1:00 P.M. one afternoon a 180-foot statue casts a shadow that is 85 feet long. Write an equation to find the angle of elevation of the Sun at that time. Find the angle of elevation. **$\tan \theta = \frac{180}{85}$; 64.7°**

3. A conveyor belt is set up to carry packages from the ground into a window 28 feet above the ground. The angle that the conveyor belt forms with the ground is 35°. How long is the conveyor belt from the ground to the window sill? **48.8 ft**

SPORTS **The distance a golf ball travels can be found using the formula $d = \frac{v_0^2}{g}\sin 2\theta$, where v_0 is the initial velocity of the ball, g is the acceleration due to gravity (which is 32 feet per second squared), and θ is the angle that the path of the ball makes with the ground.**

4. How far will a ball travel hit 90 feet per second at an angle of 55°? **237.9 ft**

5. If a ball that traveled 300 feet had an initial velocity of 110 feet per second, what angle did the path of the ball make with the ground? **26.3° or 63.7°**

6. Some children set up a teepee in the woods. The poles are 7 feet long from their intersection to their bases, and the children want the distance between the poles to be 4 feet at the base. How wide must the angle be between the poles? **33.2°**

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14-7 Skills Practice

Solving Trigonometric Equations

Find all solutions of each equation for the given interval.

1. $\sin \theta = \frac{\sqrt{2}}{2}, 0° \le \theta < 360°$ **45°, 135°**

2. $2 \cos \theta = -\sqrt{3}, 90° < \theta < 180°$ **150°**

3. $\tan^2 \theta = 1, 180° < \theta < 360°$ **225°, 315°**

4. $2 \sin \theta = 1, 0 \le \theta < \pi$ **$\frac{\pi}{6}, \frac{5\pi}{6}$**

5. $\sin^2 \theta + \sin \theta = 0, \pi \le \theta < 2\pi$ **$\pi, \frac{3\pi}{2}$**

6. $2 \cos^2 \theta + \cos \theta = 0, 0 \le \theta < \pi$ **$\frac{\pi}{2}, \frac{2\pi}{3}$**

Solve each equation for all values of θ if θ is measured in radians.

7. $2 \cos^2 \theta - \cos \theta = 1$
$0 + 2k\pi, \frac{2\pi}{3} + 2k\pi$, and $\frac{4\pi}{3} + 2k\pi$

8. $\sin^2 \theta - 2 \sin \theta + 1 = 0$
$\frac{\pi}{2} + 2k\pi$

9. $\sin \theta + \sin \theta \cos \theta = 0$
$k\pi$

10. $\sin^2 \theta = 1$
$\frac{\pi}{2} + k\pi$

11. $4 \cos \theta = -1 + 2 \cos \theta$
$\frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi$

12. $\tan \theta \cos \theta = \frac{1}{2}$
$\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$

Solve each equation for all values of θ if θ is measured in degrees.

13. $2 \sin \theta + 1 = 0$
$210° + k \cdot 360$ and $330° + k \cdot 360°$

14. $2 \cos \theta + \sqrt{3} = 0$
$150° + k \cdot 360$ and $210° + k \cdot 360°$

15. $\sqrt{2} \sin \theta + 1 = 0$
$225° + k \cdot 360°$ and $315° + k \cdot 360°$

16. $2 \cos^2 \theta = 1$
$45° + k \cdot 90°$

17. $4 \sin^2 \theta = 3$
$60° + k \cdot 180°$ and $120° + k \cdot 180°$

18. $\cos 2\theta = -1$
$90° + k \cdot 180°$

Solve each equation for all values of θ.

19. $3 \cos^2 \theta - \sin^2 \theta = 0$
$\frac{\pi}{3} + k\pi$ and $\frac{2\pi}{3} + k\pi$, or
$60° + k \cdot 180°$ and $120° + k \cdot 180°$

20. $\sin \theta + \sin 2\theta = 0$
$k\pi$ and $\frac{2\pi}{3} + 2k\pi$, or
$k \cdot 180°$ and $120° + k \cdot 360°$

21. $2 \sin^2 \theta = \sin \theta + 1$
$\frac{\pi}{2} + k\frac{2\pi}{3}$, or $90° + k \cdot 120°$

22. $\cos \theta + \sec \theta = 2$
$2k\pi$, or $k \cdot 360°$

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14-7 Practice

Solving Trigonometric Equations

Find all solutions of each equation for the given interval.

1. $\sin 2\theta = \cos \theta$, $90° \le \theta < 180°$ **90°, 150°**

2. $\sqrt{2}\cos \theta = \sin 2\theta$, $0° \le \theta < 360°$ **45°, 90°, 135°, 270°**

3. $\cos 4\theta = \cos 2\theta$, $180° \le \theta < 360°$ **180°, 240°, 300°**

4. $\cos \theta + \cos (90 - \theta) = 0$, $0 \le \theta < 2\pi$ $\frac{3\pi}{4}, \frac{7\pi}{4}$

5. $2 + \cos \theta = 2\sin^2 \theta$, $\pi \le \theta \le \frac{3\pi}{2}$ $\frac{4\pi}{3}, \frac{3\pi}{2}$

6. $\tan^2 \theta + \sec \theta = 1$, $\frac{\pi}{2} \le \theta < \pi$ $\frac{2\pi}{3}$

Solve each equation for all values of θ if θ is measured in radians.

7. $\cos^2 \theta = \sin^2 \theta$ $\frac{\pi}{4} + k\frac{\pi}{2}$

8. $\cot \theta = \cot^3 \theta$ $\frac{\pi}{2} + k\pi$ **and** $\frac{\pi}{4} + k\frac{\pi}{2}$

9. $\sqrt{2}\sin^3 \theta = \sin^2 \theta$ $k\pi, \frac{\pi}{4} + 2k\pi$, **and** $\frac{3\pi}{4} + 2k\pi$

10. $\cos^2 \theta \sin \theta = \sin \theta$ $k\pi$

11. $2\cos 2\theta = 1 - 2\sin^2 \theta$ $\frac{\pi}{4} + k\frac{\pi}{2}$

12. $\sec^2 \theta = 2$ $\frac{\pi}{4} + k\frac{\pi}{2}$

Solve each equation for all values of θ if θ is measured in degrees.

13. $\sin^2 \theta \cos \theta = \cos \theta$ **90° + k · 180°**

14. $\csc^2 \theta - 3\csc \theta + 2 = 0$ **30° + k · 360°, 90° + k · 360°, and 150° + k · 360°**

15. $\frac{3}{1 + \cos \theta} = 4(1 - \cos \theta)$ **60° + k · 180° and 120° + k · 180°**

16. $\sqrt{2}\cos^2 \theta = \cos^2 \theta$ **90° + k · 180° and 450° + k · 360°**

Solve each equation for all values of θ.

17. $4\sin^2 \theta = 3$ $\frac{\pi}{3} + k\pi$ **and** $\frac{2\pi}{3} + k\pi$, **or 60° + k · 180° and 120° + k · 180°**

18. $4\sin^2 \theta - 1 = 0$ $\frac{\pi}{6} + k\pi$ **and** $\frac{5\pi}{6} + k\pi$, **or 30° + k · 180° and 150° + k · 180°**

19. $2\sin^2 \theta - 3\sin \theta = -1$ $\frac{\pi}{6} + \frac{\pi k}{3}$, **or 30° + k · 60°**

20. $\cos 2\theta + \sin \theta - 1 = 0$ $k\pi$ **and** $\frac{\pi}{6} + 2k\pi$, **or k · 180° and 30° + k · 360°**

21. **WAVES** Waves are causing a buoy to float in a regular pattern in the water. The vertical position of the buoy can be described by the equation $h = 2\sin x$. Write an expression that describes the position of the buoy when its height is at its midline. **$k\pi$ or $k \cdot 180°$**

22. **ELECTRICITY** The electric current in a certain circuit with an alternating current can be described by the formula $i = 3\sin 240t$, where i is the current in amperes and t is the time in seconds. Write an expression that describes the times at which there is no current. **$0.75kt$**

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14-7 Word Problem Practice

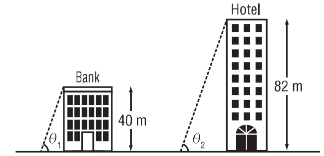
Solving Trigonometric Equations

1. **SANDCASTLES** the water level on Sunset Beach can be modeled by the function $y = 7 + 7\sin\left(\frac{\pi}{6}t\right)$, where y is the distance in feet of the shoreline above the low tide mark and t is the number of hours past 6 A.M. At 2 P.M., Victoria built her sandcastle 10 feet above the low tide mark. At what time will the shoreline reach Victoria's sandcastle? **7 P.M.**

2. **BATTERY** The amount of light emitted from a battery indicator bulb pulses while the battery is charging. This can be modeled by the equation $y = 60 + 60\sin\left(\frac{\pi}{4}t\right)$, where y is the lumens emitted from the bulb and t is the number of seconds since the beginning of a pulse. At what time will the amount of light emitted be equal to 110 lumens? **1.3 seconds after the beginning of the pulse**

BUILDINGS For Exercises 3–5, use the following information.

In downtown Centerville, the length of the shadows of the hotel and the bank depends on the angle of inclination of the sun θ.



3. Express the length of the shadow of each building as a function of the angle of inclination. **bank: $s_1 = \frac{40}{\tan \theta_1}$, hotel: $s_2 = \frac{82}{\tan \theta_2}$**

4. What is the greatest angle of inclination of the sun such that the bank is entirely contained in the shadow of the hotel? **58.6°**

5. At what angle of inclination of the sun will the bank's shadow be equal to the height of the hotel? **26°**