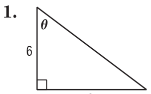
NAME ______________________ DATE ____________ PERIOD _____

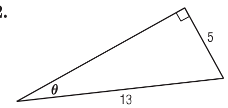
13-1 Skills Practice

Right Triangle Trigonometry

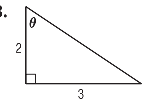
Find the values of the six trigonometric functions for angle θ.

1. 

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

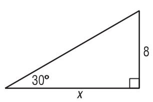
2. 

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\csc\theta = \frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = \frac{12}{5}$

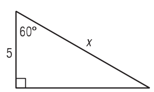
3. 

$\sin\theta = \frac{3\sqrt{13}}{13}$, $\cos\theta = \frac{2\sqrt{13}}{13}$, $\tan\theta = \frac{3}{2}$, $\csc\theta = \frac{\sqrt{13}}{3}$, $\sec\theta = \frac{\sqrt{13}}{2}$, $\cot\theta = \frac{2}{3}$

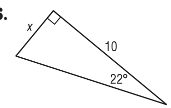
Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. 

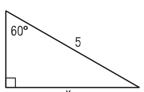
$\tan 30° = \frac{8}{x}$, $x \approx 13.9$

5. 

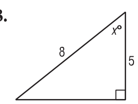
$\cos 60° = \frac{5}{x}$, $x = 10$

6. 

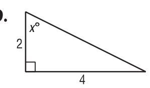
$\tan 22° = \frac{x}{10}$, $x \approx 4.0$

7. 

$\sin 60° = \frac{x}{5}$, $x \approx 4.3$

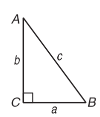
8. 

$\cos x° = \frac{5}{8}$, $x \approx 51$

9. 

$\tan x° = \frac{4}{2}$, $x \approx 63$

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 72°$, $c = 10$
$a \approx 9.5$, $b \approx 3.1$, $B = 18°$

11. $B = 20°$, $b = 15$
$a \approx 41.2$, $c \approx 43.9$, $A = 70°$

12. $A = 80°$, $a = 9$
$b \approx 1.6$, $c \approx 9.1$, $B = 10°$

13. $A = 58°$, $b = 12$
$a \approx 19.2$, $c \approx 22.6$, $B = 32°$

14. $b = 4$, $c = 9$
$a \approx 8.1$, $A \approx 64°$, $B \approx 26°$

15. $a = 7$, $b = 5$
$c \approx 8.6$, $A \approx 54°$, $B \approx 36°$

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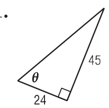
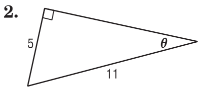
13-1 Practice

Right Triangle Trigonometry

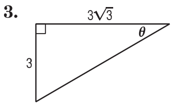
Find the values of the six trigonometric functions for angle θ.

1. 

$\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$, $\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$

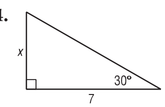
2. 

$\sin\theta = \frac{5}{11}$, $\cos\theta = \frac{4\sqrt{6}}{11}$, $\tan\theta = \frac{5\sqrt{6}}{24}$, $\csc\theta = \frac{11}{5}$, $\sec\theta = \frac{11\sqrt{6}}{24}$, $\cot\theta = \frac{4\sqrt{6}}{5}$

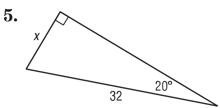
3. 

$\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = \frac{\sqrt{3}}{3}$, $\csc\theta = 2$, $\sec\theta = \frac{2\sqrt{3}}{3}$, $\cot\theta = \sqrt{3}$

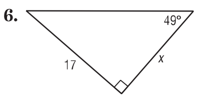
Write an equation involving sin, cos, or tan that can be used to find x. Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. 

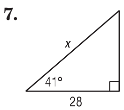
$\tan 30° = \frac{x}{7}$, $x \approx 4.0$

5. 

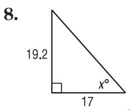
$\sin 20° = \frac{x}{32}$, $x \approx 10.9$

6. 

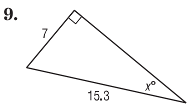
$\tan 49° = \frac{17}{x}$, $x \approx 14.8$

7. 

$\cos 41° = \frac{28}{x}$, $x \approx 37.1$

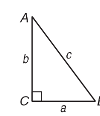
8. 

$\tan x° = \frac{19.2}{17}$, $x \approx 48$

9. 

$\sin x° = \frac{7}{15.3}$, $x \approx 27$

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



10. $A = 35°$, $a = 12$
$b \approx 17.1$, $c \approx 20.9$, $B = 55°$

11. $B = 71°$, $b = 25$
$a \approx 8.6$, $c \approx 26.4$, $A = 19°$

12. $B = 36°$, $c = 8$
$a \approx 6.5$, $b \approx 4.7$, $A = 54°$

13. $a = 4$, $b = 7$
$c \approx 8.1$, $A \approx 30°$, $B \approx 60°$

14. $A = 17°$, $c = 3.2$
$a \approx 0.9$, $b \approx 3.1$, $B = 73°$

15. $b = 52$, $c = 95$
$a \approx 79.5$, $A \approx 33°$, $B \approx 57°$

16. **SURVEYING** John stands 150 meters from a water tower and sights the top at an angle of elevation of 36°. How tall is the tower? Round to the nearest meter. **109 m**

Lesson 13-1

Answers

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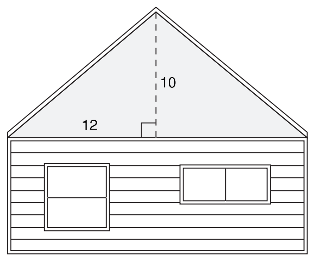
13-1 Word Problem Practice

Right Triangle Trigonometry

BUILDING For Exercises 1–4, use the following information.

The roof on a house is built with a pitch of 10/12, meaning that the roof rises 10 feet for every 12 feet of horizontal run. The side view of the roof is shown in the figure below.



1. What is the angle at the base of the roof? **39.8°**

2. What is the angle at the peak of the roof? **100.4°**

3. What is the length of the roof? **15.6 ft**

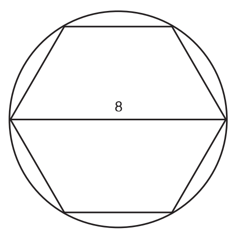
4. If the width of the house is 26 feet, what is the area of the roof? **811.2 ft^2**

5. **SLEDDING** A hill for sled riding in Blake Hills Metro Park has an angle of elevation of 26°. If the hill has a vertical drop of 225 feet, find the length of the sled run. **513.3 ft**

6. **SCALE DRAWING** The collection pool for a fountain is in the shape of a right triangle. A scale drawing shows that the angles of the triangle are 40°, 50°, and 90°. If the hypotenuse of the actual fountain will be 30 feet, what are the lengths of the other two sides of the fountain? **19.3 ft and 23.0 ft**

GEOMETRY For Exercises 7 and 8, use the following information.

A regular hexagon is inscribed in a circle with a diameter of 8 inches.



7. What is the perimeter of the hexagon? **24 in.**

8. What is the area of the hexagon? **$24\sqrt{3}$ or about 41.6 in2**

Chapter 13 10 Glencoe Algebra 2

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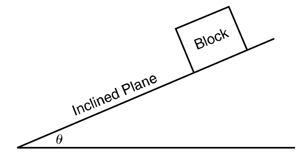
13-1 Enrichment

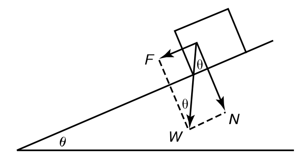
The Angle of Repose

Suppose you place a block of wood on an inclined plane, as shown at the right. If the angle, θ, at which the plane is inclined from the horizontal is very small, the block will not move. If you increase the angle, the block will eventually overcome the force of friction and start to slide down the plane.



At the instant the block begins to slide, the angle formed by the plane is called the angle of friction, or the angle of repose.

For situations in which the block and plane are smooth but unlubricated, the angle of repose depends *only* on the types of materials in the block and the plane. The angle is independent of the area of contact between the two surfaces and of the weight of the block.



The drawing at the right shows how to use vectors to find a coefficient of friction. This coefficient varies with different materials and is denoted by the Greek letter mu, μ.

$F = W \sin \theta \qquad N = W \cos$

$F = \mu N$

$\mu = \frac{\sin \theta}{\cos \theta} = \tan \theta$

Solve each problem.

Material	Coefficient of Friction μ
Wood on wood	0.5
Wood on stone	0.5
Rubber tire on dry concrete	1.0
Rubber tire on wet concrete	0.7

1. A wooden chute is built so that wooden crates can slide down into the basement of a store. What angle should the chute make in order for the crates to slide down at a constant speed? **27°**

2. Will a 100-pound wooden crate slide down a stone ramp that makes an angle of 20° with the horizontal? Explain your answer. **No, the angle must be at least 27°.**

3. If you increase the weight of the crate in Exercise 2 to 300 pounds, does it change your answer? **No, the weight does not affect the angle.**

4. A car with rubber tires is being driven on dry concrete pavement. If the car tires spin without traction on a hill, how steep is the hill? **at least 45°**

5. For Exercise 4, does it make a difference if it starts to rain? Explain your answer. **Yes, the street needs to be only 35° for the car tires to spin.**

Chapter 13 11 Glencoe Algebra 2

Lesson 13-1

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13-2 Lesson Reading Guide

Angles and Angle Measure

Pre-Activity **How can angles be used to describe circular motion?**

Read the introduction to Lesson 13-2 in your textbook.

If a gondola revolves through a complete revolution in one minute, what is its angular velocity in degrees per second? **6° per second**

Reading the Lesson

1. Match each degree measure with the corresponding radian measure on the right.

a. 30° **v**	**i.** $\frac{2\pi}{3}$
b. 90° **ii**	**ii.** $\frac{\pi}{2}$
c. 120° **i**	**iii.** $\frac{7\pi}{6}$
d. 135° **vi**	**iv.** π
e. 180° **iv**	**v.** $\frac{\pi}{6}$
f. 210° **iii**	**vi.** $\frac{3\pi}{4}$

2. The sine of 30° is $\frac{1}{2}$ and the sine of 150° is also $\frac{1}{2}$. Does this mean that 30° and 150° are coterminal angles? Explain your reasoning. **Sample answer: No; the terminal side of a 30° angle is in Quadrant I, while the terminal side of a 150° angle is in Quadrant II.**

3. Describe how to find two angles that are coterminal with an angle of 155°, one with positive measure and one with negative measure. (Do not actually calculate these angles.) **Sample answer: Positive angle: Add 360° to 155°. Negative angle: Subtract 360° from 155°.**

4. Describe how to find two angles that are coterminal with an angle of $\frac{5\pi}{3}$, one positive and one negative. (Do not actually calculate these angles.) **Sample answer: Positive angle: Add 2π to $\frac{5\pi}{3}$. Negative angle: Subtract 2π from $\frac{5\pi}{3}$.**

Helping You Remember

5. How can you use what you know about the circumference of a circle to remember how to convert between radian and degree measure? **Sample answer: The circumference of a circle is given by the formula $C = 2\pi r$, so the circumference of a circle with radius 1 is 2π. In degree measure, one complete circle is 360°. So 2π radians = 360° and π radians = 180°.**

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13-2 Study Guide and Intervention

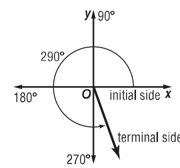
Angles and Angle Measurement

Angle Measurement An angle is determined by two rays. The degree measure of an angle is described by the amount and direction of rotation from the **initial side** along the positive x-axis to the **terminal side**. A counterclockwise rotation is associated with positive angle measure and a clockwise rotation is associated with negative angle measure. An angle can also be measured in **radians**.

Radian and Degree Measure	To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$. To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.

Example 1 **Draw an angle with measure 290° in standard notation.**

The negative y-axis represents a positive rotation of 270°. To generate an angle of 290°, rotate the terminal side 20° more in the counterclockwise direction.



Example 2 **Rewrite the degree measure in radians and the radian measure in degrees.**

a. 45°

$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4}$ radians

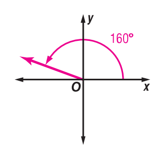
b. $\frac{5\pi}{3}$ radians

$\frac{5\pi}{3}$ radians $= \frac{5\pi}{3}\left(\frac{180°}{\pi}\right) = 300°$

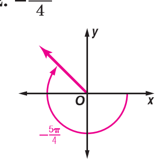
Exercises

Draw an angle with the given measure in standard position.

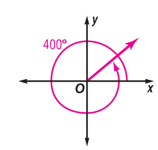
1. 160°



2. $-\frac{5\pi}{4}$



3. 400°



Rewrite each degree measure in radians and each radian measure in degrees.

4. 140°	**5.** −860°	**6.** $-\frac{3\pi}{5}$	**7.** $\frac{11\pi}{3}$
$\frac{7\pi}{9}$	$-\frac{43\pi}{9}$	−108°	660°

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13-2 Study Guide and Intervention *(continued)*

Angles and Angle Measurement

Coterminal Angles When two angles in standard position have the same terminal sides, they are called **coterminal angles**. You can find an angle that is coterminal to a given angle by adding or subtracting a multiple of 360°. In radian measure, a coterminal angle is found by adding or subtracting a multiple of 2π.

Example **Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

a. 250°

A positive angle is $250° + 360°$ or $610°$.

A negative angle is $250° - 360°$ or $-110°$.

b. $\frac{5\pi}{8}$

A positive angle is $\frac{5\pi}{8} + 2\pi$ or $\frac{21\pi}{8}$.

A negative angle is $\frac{5\pi}{8} - 2\pi$ or $-\frac{11\pi}{8}$.

Exercises

Find one angle with a positive measure and one angle with a negative measure coterminal with each angle. **1–18 Sample answers are given.**

1. 65°
425°, −295°

2. −75°
285°, −435°

3. 230°
590°, −130°

4. 420°
60°, −300°

5. 340°
700°, −20°

6. −130°
230°, −490°

7. −290°
70°, −650°

8. 690°
330°, −30°

9. −420°
300°, −60°

10. $\frac{\pi}{9}$
$\frac{19\pi}{9}, -\frac{17\pi}{9}$

11. $\frac{3\pi}{8}$
$\frac{19\pi}{8}, -\frac{13\pi}{8}$

12. $\frac{6\pi}{5}$
$\frac{16\pi}{5}, -\frac{4\pi}{5}$

13. $\frac{-7\pi}{4}$
$\frac{\pi}{4}, -\frac{15\pi}{4}$

14. $\frac{15\pi}{4}$
$\frac{7\pi}{4}, -\frac{\pi}{4}$

15. $\frac{-13\pi}{6}$
$\frac{11\pi}{6}, -\frac{\pi}{6}$

16. $\frac{17\pi}{5}$
$\frac{7\pi}{5}, -\frac{3\pi}{5}$

17. $\frac{-5\pi}{3}$
$\frac{\pi}{3}, -\frac{11\pi}{3}$

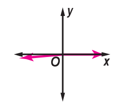
18. $\frac{-11\pi}{4}$
$\frac{5\pi}{4}, -\frac{3\pi}{4}$

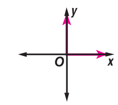
NAME ______________________ DATE __________ PERIOD _____

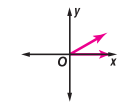
13-2 Skills Practice

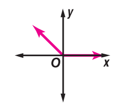
Angles and Angle Measure

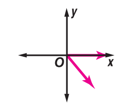
Draw an angle with the given measure in standard position.

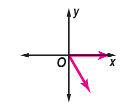
1. 185°


2. 810°


3. 390°


4. 495°


5. −50°


6. −420°


Rewrite each degree measure in radians and each radian measure in degrees.

7. 130° $\frac{13\pi}{18}$

8. 720° 4π

9. 210° $\frac{7\pi}{6}$

10. 90° $\frac{\pi}{2}$

11. −30° $-\frac{\pi}{6}$

12. −270° $-\frac{3\pi}{2}$

13. $\frac{\pi}{3}$ **60°**

14. $\frac{5\pi}{6}$ **150°**

15. $\frac{2\pi}{3}$ **120°**

16. $\frac{5\pi}{4}$ **225°**

17. $-\frac{3\pi}{4}$ **−135°**

18. $-\frac{7\pi}{6}$ **−210°**

Find one angle with positive measure and one angle with negative measure coterminal with each angle. **19–26. Sample answers are given.**

19. 45° **405°, −315°**

20. 60° **420°, −300°**

21. 370° **10°, −350°**

22. −90° **270°, −450°**

23. $\frac{2\pi}{3}$ $\frac{8\pi}{3}, -\frac{4\pi}{3}$

24. $\frac{5\pi}{2}$ $\frac{9\pi}{2}, -\frac{3\pi}{2}$

25. $\frac{\pi}{6}$ $\frac{13\pi}{6}, -\frac{11\pi}{6}$

26. $-\frac{3\pi}{4}$ $\frac{5\pi}{4}, -\frac{11\pi}{4}$

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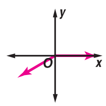
13-2 Practice (Average)

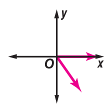
Angles and Angle Measure

Draw an angle with the given measure in standard position.

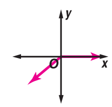
1. 210°



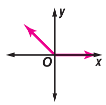
2. 305°



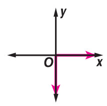
3. 580°



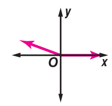
4. 135°



5. −450°



6. −560°



Rewrite each degree measure in radians and each radian measure in degrees.

7. 18° $\frac{\pi}{10}$
8. 6° $\frac{\pi}{30}$
9. 870° $\frac{29\pi}{6}$
10. 347° $\frac{347\pi}{180}$

11. −72° $-\frac{2\pi}{5}$
12. −820° $-\frac{41\pi}{9}$
13. −250° $-\frac{25\pi}{18}$
14. −165° $-\frac{11\pi}{12}$

15. 4π 720°
16. $\frac{5\pi}{2}$ 450°
17. $\frac{13\pi}{5}$ 468°
18. $\frac{13\pi}{30}$ 78°

19. $-\frac{9\pi}{2}$ −810°
20. $-\frac{7\pi}{12}$ −105°
21. $-\frac{3\pi}{8}$ −67.5°
22. $-\frac{3\pi}{16}$ −33.75°

Find one angle with positive measure and one angle with negative measure coterminal with each angle. 23–34. Sample answers are given.

23. 65° 425°, −295°
24. 80° 440°, −280°
25. 285° 645°, −75°

26. 110° 470°, −250°
27. −37° 323°, −397°
28. −93° 267°, −453°

29. $\frac{2\pi}{5}$ $\frac{12\pi}{5}$, $-\frac{8\pi}{5}$
30. $\frac{5\pi}{6}$ $\frac{17\pi}{6}$, $-\frac{7\pi}{6}$
31. $\frac{17\pi}{6}$ $\frac{29\pi}{6}$, $-\frac{7\pi}{6}$

32. $-\frac{3\pi}{2}$ $\frac{\pi}{2}$, $-\frac{7\pi}{2}$
33. $-\frac{\pi}{4}$ $\frac{7\pi}{4}$, $-\frac{9\pi}{4}$
34. $-\frac{5\pi}{12}$ $\frac{19\pi}{12}$, $-\frac{29\pi}{12}$

35. TIME Find both the degree and radian measures of the angle through which the hour hand on a clock rotates from 5 A.M. to 10 A.M. −150°; $-\frac{5\pi}{6}$

36. ROTATION A truck with 16-inch radius wheels is driven at 77 feet per second (52.5 miles per hour). Find the measure of the angle through which a point on the outside of the wheel travels each second. Round to the nearest degree and nearest radian. 3309°/s; 58 radians/s

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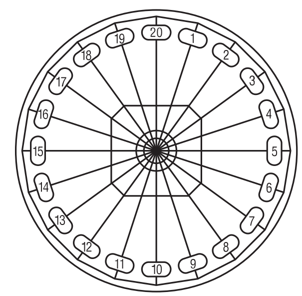
13-2 Word Problem Practice

Angles and Angle Measure

AMUSEMENT PARKS For Exercises 1–4, use the following information.
The carousel at an amusement park has 20 horses spaced evenly around its circumference. The horses are numbered consecutively from 1 to 20. The carousel completes one rotation about its axis every 40 seconds.



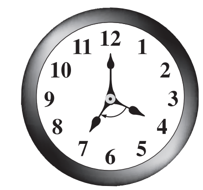
1. What is the central angle, in degrees, formed by horse #1 and horse #8? 126°

2. What is the speed of the carousel in rotations per minute? 1.5 rotations per minute

3. What is the speed of the carousel in radians per minute? 3π radians per minute

4. A child rides the carousel for 6 minutes. Through how many radians will the child pass in the course of the carousel ride? 18 π radians

5. TIME Through what angle, in degrees and radians, does the hour hand on a clock rotate between 4 P.M. and 7 P.M.? 90°, $\frac{\pi}{2}$ radians



6. TIME Through what angle, in degrees and radians, does the minute hand rotate between 4 P.M. and 7 P.M.? 1080° 6π radians

7. PLANETS Earth makes one full rotation on its axis every 24 hours. How long does it take Earth to rotate through 150°? Neptune makes one full rotation on its axis every 16 hours. How long does it take Saturn to rotate through 150°?

10 hours, $6\frac{2}{3}$ hours

Lesson 13-2

Answers

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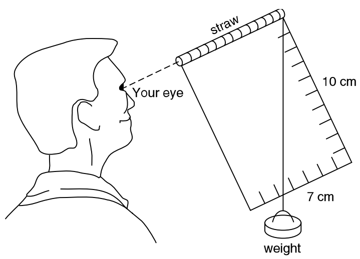
13-2 Enrichment

Making and Using a Hypsometer

A **hypsometer** is a device that can be used to measure the height of an object. To construct your own hypsometer, you will need a rectangular piece of heavy cardboard that is at least 7 cm by 10 cm, a straw, transparent tape, a string about 20 cm long, and a small weight that can be attached to the string.

Mark off 1-cm increments along one short side and one long side of the cardboard. Tape the straw to the other short side. Then attach the weight to one end of the string, and attach the other end of the string to one corner of the cardboard, as shown in the figure below. The diagram below shows how your hypsometer should look.



To use the hypsometer, you will need to measure the distance from the base of the object whose height you are finding to where you stand when you use the hypsometer.

Sight the top of the object through the straw. Note where the free-hanging string crosses the bottom scale. Then use similar triangles to find the height of the object.

1. Draw a diagram to illustrate how you can use similar triangles and the hypsometer to find the height of a tall object. **See students' diagrams.**

Use your hypsometer to find the height of each of the following.

2. your school's flagpole **See students' work.**

3. a tree on your school's property

4. the highest point on the front wall of your school building

5. the goal posts on a football field

6. the hoop on a basketball court

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13-3 Lesson Reading Guide

Trigonometric Functions of General Angles

Pre-Activity **How can you model the position of riders on a skycoaster?**

Read the introduction to Lesson 13-3 in your textbook.

- What does $t = 0$ represent in this application? **Sample answer: the time when the riders leave the bottom of their swing**
- Do negative values of t make sense in this application? Explain your answer. **Sample answer: No; $t = 0$ represents the starting time, so the value of t cannot be less than 0.**

Reading the Lesson

1. Suppose θ is an angle in standard position, $P(x, y)$ is a point on the terminal side of θ, and the distance from the origin to P is r. Determine whether each of the following statements is *true* or *false*.

 a. The value of r can be found by using either the Pythagorean Theorem or the distance formula. **true**

 b. $\cos \theta = \frac{x}{r}$ **true**

 c. $\csc \theta$ is defined if $y \neq 0$. **true**

 d. $\tan \theta$ is undefined if $y = 0$. **false**

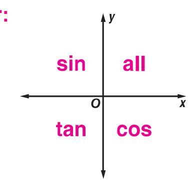
 e. $\sin \theta$ is defined for every value of θ. **true**

2. Let θ be an angle measured in degrees. Match the quadrant of θ from the first column with the description of how to find the reference angle for θ from the second column.

a. Quadrant III **ii**	i. Subtract θ from 360°.
b. Quadrant IV **i**	ii. Subtract 180° from θ.
c. Quadrant II **iv**	iii. θ is its own reference angle.
d. Quadrant I **iii**	iv. Subtract θ from 180°.

Helping You Remember

3. The chart on page 779 in your textbook summarizes the signs of the six trigonometric functions in the four quadrants. Since reciprocals always have the same sign, you only need to remember where the sine, cosine, and tangent are positive. How can you remember this with a simple diagram?
 Sample answer:



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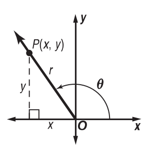
13-3 Study Guide and Intervention

Trigonometric Functions of General Angles

Trigonometric Functions and General Angles

Trigonometric Functions, θ in Standard Position



Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. By the Pythagorean Theorem, the distance r from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\csc \theta = \frac{r}{y}$ $\sec \theta = \frac{r}{x}$ $\cot \theta = \frac{x}{y}$

Example **Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point $(-5, 5\sqrt{2})$.**

You know that $x = -5$ and $y = 5\sqrt{2}$. You need to find r.

$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$= \sqrt{(-5)^2 + (5\sqrt{2})^2}$ Replace x with -5 and y with $5\sqrt{2}$.

$= \sqrt{75}$ or $5\sqrt{3}$

Now use $x = -5$, $y = 5\sqrt{2}$, and $r = 5\sqrt{3}$ to write the ratios.

$\sin \theta = \frac{y}{r} = \frac{5\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{6}}{3}$ $\cos \theta = \frac{x}{r} = \frac{-5}{5\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\tan \theta = \frac{y}{x} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$

$\csc \theta = \frac{r}{y} = \frac{5\sqrt{3}}{5\sqrt{2}} = \frac{\sqrt{6}}{2}$ $\sec \theta = \frac{r}{x} = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$ $\cot \theta = \frac{x}{y} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Exercises

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (8, 4)

$\sin \theta = \frac{\sqrt{5}}{5}$, $\cos \theta = \frac{2\sqrt{5}}{5}$, $\tan \theta = \frac{1}{2}$, $\csc \theta = \sqrt{5}$, $\sec \theta = \frac{\sqrt{5}}{2}$, $\cot \theta = 2$

2. $(4, 4\sqrt{3})$

$\sin \theta = \frac{\sqrt{3}}{2}$, $\cos \theta = \frac{1}{2}$, $\tan \theta = \sqrt{3}$, $\csc \theta = \frac{2\sqrt{3}}{3}$, $\sec \theta = 2$, $\cot \theta = \frac{\sqrt{3}}{3}$

3. (0, −4)

$\sin \theta = -1$, $\cos \theta = 0$, $\tan \theta$ undefined, $\csc \theta = -1$, $\sec \theta$ undefined, $\cot \theta = 0$

4. (6, 2)

$\sin \theta = \frac{\sqrt{10}}{10}$, $\cos \theta = \frac{3\sqrt{10}}{10}$, $\tan \theta = \frac{1}{3}$, $\csc \theta = \sqrt{10}$, $\sec \theta = \frac{\sqrt{10}}{3}$, $\cot \theta = 3$

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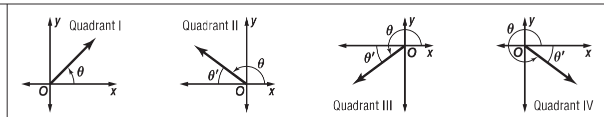
13-3 Study Guide and Intervention *(continued)*

Trigonometric Functions of General Angles

Reference Angles If θ is a nonquadrantal angle in standard position, its reference angle θ' is defined as the acute angle formed by the terminal side of θ and the x-axis.

Reference Angle Rule



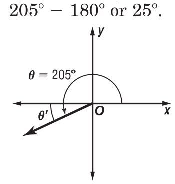
Quadrant I	Quadrant II	Quadrant III	Quadrant IV
$\theta' = \theta$	$\theta' = 180° - \theta$ $(\theta' = \pi - \theta)$	$\theta' = \theta - 180°$ $(\theta' = \theta - \pi)$	$\theta' = 360° - \theta$ $(\theta' = 2\pi - \theta)$

Signs of Trigonometric Functions

Function	I	II	III	IV
$\sin \theta$ or $\csc \theta$	+	+	−	−
$\cos \theta$ or $\sec \theta$	+	−	−	+
$\tan \theta$ or $\cot \theta$	+	−	+	−

Example 1 **Sketch an angle of measure 205°. Then find its reference angle.**

Because the terminal side of 205° lies in Quadrant III, the reference angle θ' is $205° - 180°$ or 25°.



Example 2 **Use a reference angle to find the exact value of $\cos \frac{3\pi}{4}$.**

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.

The cosine function is negative in Quadrant II.

$\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

Exercises

Find the exact value of each trigonometric function.

1. $\tan(-510°)$ **$\frac{\sqrt{3}}{3}$**

2. $\csc \frac{11\pi}{4}$ **$\sqrt{2}$**

3. $\sin(-90°)$ **−1**

4. $\cot 1665°$ **1**

5. $\cot 30°$ **$\sqrt{3}$**

6. $\tan 315°$ **−1**

7. $\csc \frac{\pi}{4}$ **$\sqrt{2}$**

8. $\tan \frac{4\pi}{3}$ **$\sqrt{3}$**

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13-3 Skills Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (5, 12)
$\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = \frac{5}{12}$

2. (3, 4)
$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

3. (8, −15)
$\sin\theta = -\frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = -\frac{15}{8}$, $\csc\theta = -\frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = -\frac{8}{15}$

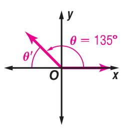
4. (−4, 3)
$\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

5. (−9, −40)
$\sin\theta = -\frac{40}{41}$, $\cos\theta = -\frac{9}{41}$, $\tan\theta = \frac{40}{9}$, $\csc\theta = -\frac{41}{40}$, $\sec\theta = -\frac{41}{9}$, $\cot\theta = \frac{9}{40}$

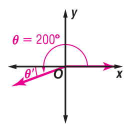
6. (1, 2)
$\sin\theta = \frac{2\sqrt{5}}{5}$, $\cos\theta = \frac{\sqrt{5}}{5}$, $\tan\theta = 2$, $\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = \sqrt{5}$, $\cot\theta = \frac{1}{2}$

Sketch each angle. Then find its reference angle.

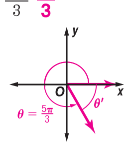
7. 135° **45°**



8. 200° **20°**



9. $\frac{5\pi}{3}$ $\frac{\pi}{3}$



Find the exact value of each trigonometric function.

10. sin 150° $\frac{1}{2}$ **11.** cos 270° **0** **12.** cot 135° **−1** **13.** tan (−30°) $-\frac{\sqrt{3}}{3}$

14. $\tan\frac{\pi}{4}$ **1** **15.** $\cos\frac{4\pi}{3}$ $-\frac{1}{2}$ **16.** cot (−π) **undefined** **17.** $\sin\left(-\frac{3\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

18. $\sin\theta = \frac{4}{5}$, Quadrant II
$\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

19. $\tan\theta = -\frac{12}{5}$, Quadrant IV
$\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\csc\theta = -\frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$

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13-3 Practice

Trigonometric Functions of General Angles

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. (6, 8)
$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

2. (−20, 21)
$\sin\theta = \frac{21}{29}$, $\cos\theta = -\frac{20}{29}$, $\tan\theta = -\frac{21}{20}$, $\csc\theta = \frac{29}{21}$, $\sec\theta = -\frac{29}{20}$, $\cot\theta = -\frac{20}{21}$

3. (−2, −5)
$\sin\theta = -\frac{5\sqrt{29}}{29}$, $\cos\theta = -\frac{2\sqrt{29}}{29}$, $\tan\theta = \frac{5}{2}$, $\csc\theta = -\frac{\sqrt{29}}{5}$, $\sec\theta = -\frac{\sqrt{29}}{2}$, $\cot\theta = \frac{2}{5}$

Find the reference angle for the angle with the given measure.

4. 236° **56°** **5.** $\frac{13\pi}{8}$ $\frac{3\pi}{8}$ **6.** −210° **30°** **7.** $-\frac{7\pi}{4}$ $\frac{\pi}{4}$

Find the exact value of each trigonometric function.

8. tan 135° **−1** **9.** cot 210° $\sqrt{3}$ **10.** cot (−90°) **0** **11.** cos 405° $\frac{\sqrt{2}}{2}$

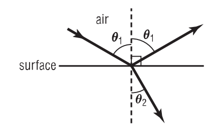
12. $\tan\frac{5\pi}{3}$ $-\sqrt{3}$ **13.** $\csc\left(-\frac{3\pi}{4}\right)$ $-\sqrt{2}$ **14.** cot 2π **undefined** **15.** $\tan\frac{13\pi}{6}$ $\frac{\sqrt{3}}{3}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

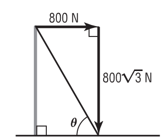
16. $\tan\theta = -\frac{12}{5}$, Quadrant IV
$\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\csc\theta = -\frac{13}{12}$, $\sec\theta = \frac{13}{5}$, $\cot\theta = -\frac{5}{12}$

17. $\sin\theta = \frac{2}{3}$, Quadrant III
$\cos\theta = -\frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{2\sqrt{5}}{5}$, $\csc\theta = \frac{3}{2}$, $\sec\theta = -\frac{3\sqrt{5}}{5}$, $\cot\theta = -\frac{\sqrt{5}}{2}$

18. LIGHT Light rays that "bounce off" a surface are *reflected* by the surface. If the surface is partially transparent, some of the light rays are bent or *refracted* as they pass from the air through the material. The angles of reflection θ_1 and of refraction θ_2 in the diagram at the right are related by the equation $\sin\theta_1 = n\sin\theta_2$. If $\theta_1 = 60°$ and $n = \sqrt{3}$, find the measure of θ_2. **30°**



19. FORCE A cable running from the top of a utility pole to the ground exerts a horizontal pull of 800 Newtons and a vertical pull of $800\sqrt{3}$ Newtons. What is the sine of the angle θ between the cable and the ground? What is the measure of this angle? $\frac{\sqrt{3}}{2}$; **60°**



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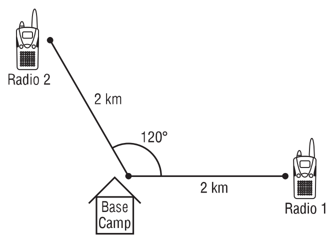
13-3 Word Problem Practice

Trigonometric Functions of General Angles

1. **RADIOS** Two correspondence radios are located 2 kilometers away from a base camp. The angle formed between the first radio, the base camp, and the second radio is 120°. If the first radio has coordinates (2, 0) relative to the base camp, what is the position of the second radio relative to the base camp?

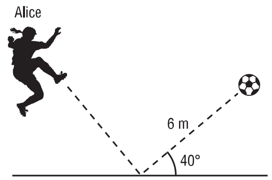


$(-1, \sqrt{2})$

2. **CLOCKS** The pendulum of a grandfather clock swings back and forth through an arc. The angle θ of the pendulum is given by $\theta = 0.3 \cos\left(\frac{\pi}{2} + 5t\right)$ where t is the time in seconds after leaving the bottom of the swing. Determine the measure of the angles for t + 0, 0.5, 1, 1.5, 2, 2.5, and 3 in radians. **0, −0.18, 0.28, −0.28, 0.16, 0.02, −0.20**

3. **CARNIVALS** Janice rides a Ferris wheel that is 30 meters in diameter. When Janice gets in her seat at the bottom of the ride she is 1.5 meters from the ground. How far from the ground will she be after the Ferris wheel rotates 225°? **27.1 m**

4. **SOCCER** Alice kicks a soccer ball towards a wall. The ball is deflected off the wall at an angle of 40°, and it rolls 6 meters. How far is the soccer ball from the wall when it stops rolling? **3.9 m**



PAPER AIRPLANES For Exercise 5 and 6, use the following information.

The formula $R = \frac{V_0^2 \sin 2\theta}{32} + 15 \cos \theta$ gives the distance traveled by a paper airplane that is thrown with an initial velocity of V_0 feet per second at an angle of θ with the ground.

5. If the airplane is thrown with an initial velocity of 15 feet per second at an angle of 25°, how far will the airplane travel? **19 ft**

6. Two airplanes are thrown with an initial velocity of 10 feet per second. One airplane is thrown at an angle of 15° to the ground, and the other airplane is thrown at an angle of 45° to the ground. Which will travel further? **The airplane thrown at 15° will travel further.**

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13-3 Enrichment

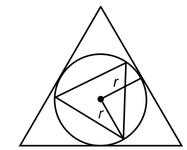
Areas of Polygons and Circles

A regular polygon has sides of equal length and angles of equal measure. A regular polygon can be inscribed in or circumscribed about a circle. For n-sided regular polygons, the following area formulas can be used.

Area of circle $A_C = \pi r^2$

Area of inscribed polygon $A_I = \frac{nr^2}{2} \times \sin \frac{360°}{n}$

Area of circumscribed polygon $A_C = nr^2 \times \tan \frac{180°}{n}$



Use a calculator to complete the chart below for a unit circle (a circle of radius 1).

	Number of Sides	Area of Inscribed Polygon	Area of Circle minus Area of Polygon	Area of Circumscribed Polygon	Area of Polygon minus Area of Circle
	3	1.2990381	1.8425545	5.1961524	2.054597
1.	4	**2**	**1.1415927**	**4**	**0.8584073**
2.	8	**2.8284271**	**0.3131655**	**3.3137085**	**0.1721158**
3.	12	**3**	**0.1415926**	**3.2153903**	**0.0737977**
4.	20	**3.0901699**	**0.0514227**	**3.1676888**	**0.0260961**
5.	24	**3.1058285**	**0.0357641**	**3.1596599**	**0.0180672**
6.	28	**3.1152931**	**0.0262996**	**3.1548423**	**0.0132496**
7.	32	**3.1214452**	**0.0201475**	**3.1517249**	**0.0101322**
8.	1000	**3.1415720**	**0.0000206**	**3.1416030**	**0.0000103**

9. What number do the areas of the circumscribed and inscribed polygons seem to be approaching? **π**

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13-4 Lesson Reading Guide

Law of Sines

Pre-Activity **How can trigonometry be used to find the area of a triangle?**

Read the introduction to Lesson 13-4 in your textbook.

What happens when the formula Area $= \frac{1}{2}ab \sin C$ is applied to a right triangle in which C is the right angle? **Sample answer: The formula gives Area $= \frac{1}{2}ab \sin 90° = \frac{1}{2}ab \cdot 1 = \frac{1}{2}ab$, which is the same as the result from using the formula Area $= \frac{1}{2}$(base)(height).**

Reading the Lesson

1. In each case below, the measures of three parts of a triangle are given. For each case, write the formula you would use to find the area of the triangle. Show the formulas with specific values substituted, but do not actually calculate the area. If there is not enough information provided to find the area of the triangle by using the area formulas on page 725 in your textbook and without finding other parts of the triangle first, explain why.

   a. $A = 48°, b = 9, c = 5$ **$\frac{1}{2}(9)(5) \sin 48°$**

   b. $a = 15, b = 15, C = 120°$ **$\frac{1}{2}(15)(15) \sin 120°$**

   c. $b = 16, c = 10, B = 120°$ **Not enough information; *B* is not the included angle between the two given sides.**

2. Tell whether the equation must be true based on the Law of Sines. Write *yes* or *no*.

   a. $\frac{\sin A}{b} = \frac{\sin B}{a}$ **no**

   b. $\frac{b}{\sin B} = \frac{c}{\sin C}$ **yes**

   c. $a \sin C = c \sin A$ **yes**

   d. $b = \frac{a \sin A}{\sin B}$ **no**

3. Determine whether $\triangle ABC$ has *no solution*, *one solution*, or *two solutions*. Do not try to solve the triangle.

   a. $a = 20, A = 30°, B = 70°$ **one solution**

   b. $A = 55°, b = 5, a = 3$ ($b \sin A \approx 4.1$) **no solution**

   c. $c = 12, A = 100°, a = 30$ **one solution**

   d. $C = 27°, b = 23.5, c = 17.5$ ($b \sin C \approx 10.7$) **two solutions**

Helping You Remember

4. Suppose that you are taking a quiz and cannot remember whether the formula for the area of a triangle is Area $= \frac{1}{2}ab \cos C$ or Area $= \frac{1}{2}ab \sin C$. How can you quickly remember which of these is correct? **Sample answer: The formula has to work when *C* is a right angle. The formula cannot contain cos *C* because cos 90° = 0 and this would make the area of a right triangle be 0.**

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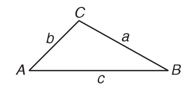
13-4 Study Guide and Intervention

Law of Sines

Law of Sines The area of any triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of a Triangle	area $= \frac{1}{2} bc \sin A$ area $= \frac{1}{2} ac \sin B$ area $= \frac{1}{2} ab \sin C$



You can use the Law of Sines to solve any triangle if you know the measures of two angles and any side, or the measures of two sides and the angle opposite one of them.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **Find the area of $\triangle ABC$ if $a = 10$, $b = 14$, and $C = 40°$.**

Area $= \frac{1}{2}ab \sin C$ — Area formula

$= \frac{1}{2}(10)(14)\sin 40°$ — Replace *a*, *b*, and *C*.

≈ 44.9951 — Use a calculator.

The area of the triangle is approximately 45 square units.

Example 2 **If $a = 12$, $b = 9$, and $A = 28°$, find B.**

$\frac{\sin A}{a} = \frac{\sin B}{b}$ — Law of Sines

$\frac{\sin 28°}{12} = \frac{\sin B}{9}$ — Replace *A*, *a*, and *b*.

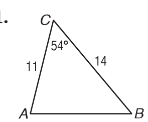
$\sin B = \frac{9 \sin 28°}{12}$ — Solve for sin *B*.

$\sin B \approx 0.3521$ — Use a calculator.

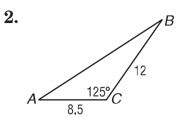
$B \approx 20.62°$ — Use the $\sin^{-1}$ function.

Exercises

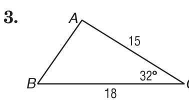
Find the area of $\triangle ABC$ to the nearest tenth.

1. 

62.3 units²

2. 

41.8 units²

3. 

71.5 units²

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

4. $B = 42°, C = 68°, a = 10$ **$A = 70°, b \approx 7.1, c \approx 9.9$**

5. $A = 40°, B = 14°, a = 52$ **$b \approx 19.6, c \approx 65.4, C = 126°$**

6. $A = 15°, B = 50°, b = 36$ **$C = 115°, a \approx 12.2, c \approx 42.6$**

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13-4 Study Guide and Intervention *(continued)*

Law of Sines

One, Two, or No Solutions

Possible Triangles Given Two Sides and One Opposite Angle	Suppose you are given *a*, *b*, and *A* for a triangle. If *a* is acute: $a < b \sin A \Rightarrow$ no solution $a = b \sin A \Rightarrow$ one solution $b > a > b \sin A \Rightarrow$ two solutions $a > b \Rightarrow$ one solution If *A* is right or obtuse: $a \le b \Rightarrow$ no solution $a > b \Rightarrow$ one solution

Example **Determine whether $\triangle ABC$ has no solutions, one solution, or two solutions. Then solve $\triangle ABC$.**

a. $A = 48°$, $a = 11$, and $b = 16$

Since *A* is acute, find $b \sin A$ and compare it with *a*.
$b \sin A = 16 \sin 48° \approx 11.89$
Since $11 < 11.89$, there is no solution.

b. $A = 34°$, $a = 6$, $b = 8$

Since *A* is acute, find $b \sin A$ and compare it with *a*; $b \sin A = 8 \sin 34° \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute *B*

First use the Law of Sines to find *B*.

$\frac{\sin B}{8} = \frac{\sin 34°}{6}$

$\sin B \approx 0.7456$

$B \approx 48°$

The measure of angle *C* is about $180° - (34° + 48°)$ or about 98°.

Use the Law of Sines again to find *c*.

$\frac{\sin 98°}{c} \approx \frac{\sin 34°}{6}$

$c \approx \frac{6 \sin 98°}{\sin 34°}$

$c \approx 10.6$

Obtuse *B*

To find *B* you need to find an obtuse angle whose sine is also 0.7456.

To do this, subtract the angle given by your calculator, 48°, from 180°. So *B* is approximately 132°.

The measure of angle *C* is about $180° - (34° + 132°)$ or about 14°.

Use the Law of Sines to find *c*.

$\frac{\sin 14°}{c} \approx \frac{\sin 34°}{6}$

$c \approx \frac{6 \sin 14°}{\sin 34°}$

$c \approx 2.6$

Exercises

Determine whether each triangle has no solutions, one solution, or two solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 50°$, $a = 34$, $b = 40$ **two solutions; $B \approx 64°$, $C \approx 66°$, $c \approx 40.5$; $B \approx 116°$, $C \approx 14°$, $c \approx 10.7$**

2. $A = 24°$, $a = 3$, $b = 8$ **no solutions**

3. $A = 125°$, $a = 22$, $b = 15$ **one solution; $B \approx 34°$, $C \approx 21°$, $c \approx 9.6$**

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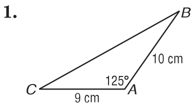
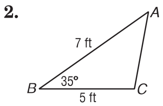
13-4 Skills Practice

Law of Sines

Find the area of $\triangle ABC$ to the nearest tenth.

1. 

36.9 cm²

2. 

10.0 ft²

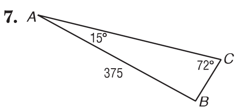
3. $A = 35°$, $b = 3$ ft, $c = 7$ ft **6.0 ft²**

4. $C = 148°$, $a = 10$ cm, $b = 7$ cm **18.5 cm²**

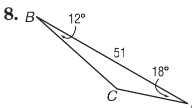
5. $C = 22°$, $a = 14$ m, $b = 8$ m **21.0 m²**

6. $B = 93°$, $c = 18$ mi, $a = 42$ mi **377.5 mi²**

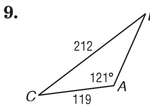
Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

7. 

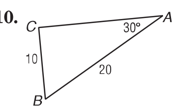
$B = 93°$, $a \approx 102.1$, $b \approx 393.8$

8. 

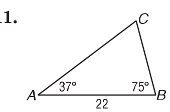
$C = 150°$, $a \approx 31.5$, $b \approx 21.2$

9. 

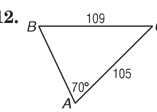
$B \approx 29°$, $C \approx 30°$, $c \approx 124.6$

10. 

$B = 60°$, $C = 90°$, $b \approx 17.3$

11. 

$C = 68°$, $a \approx 14.3$, $b \approx 22.9$

12. 

$B \approx 65°$, $C \approx 45°$, $c \approx 82.2$

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

13. $A = 30°$, $a = 1$, $b = 4$ **no solution**

14. $A = 30°$, $a = 2$, $b = 4$ **one solution; $B = 90°$, $C = 60°$, $c \approx 3.5$**

15. $A = 30°$, $a = 3$, $b = 4$ **two solutions; $B \approx 42°$, $C \approx 108°$, $c \approx 5.7$; $B \approx 138°$, $C \approx 12°$, $c \approx 1.2$**

16. $A = 38°$, $a = 10$, $b = 9$ **one solution; $B \approx 34°$, $C \approx 108°$, $c \approx 15.4$**

17. $A = 78°$, $a = 8$, $b = 5$ **one solution; $B \approx 38°$, $C \approx 64°$, $c \approx 7.4$**

18. $A = 133°$, $a = 9$, $b = 7$ **one solution; $B \approx 35°$, $C \approx 12°$, $c \approx 2.6$**

19. $A = 127°$, $a = 2$, $b = 6$ **no solution**

20. $A = 109°$, $a = 24$, $b = 13$ **one solution; $B \approx 31°$, $C \approx 40°$, $c \approx 16.4$**

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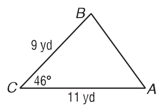
13-4 Practice

Law of Sines

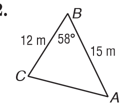
Find the area of $\triangle ABC$ to the nearest tenth.

1.



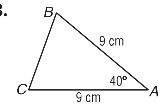
35.6 yd²

2.



76.3 m²

3.



26.0 cm²

4. $C = 32°$, $a = 12.6$ m, $b = 8.9$ m **29.7 m²**

5. $B = 27°$, $a = 14.9$ cm, $c = 18.6$ cm **62.9 cm²**

6. $A = 17.4°$, $b = 12$ km, $c = 14$ km **25.1 km²**

7. $A = 34°$, $b = 19.4$ ft, $c = 8.6$ ft **46.6 ft²**

Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

8. $A = 50°$, $B = 30°$, $c = 9$ **$C = 100°$, $a \approx 7.0$, $b \approx 4.6$**

9. $A = 56°$, $B = 38°$, $a = 12$ **$C = 86°$, $b \approx 8.9$, $c \approx 14.4$**

10. $A = 80°$, $C = 14°$, $a = 40$ **$B = 86°$, $b \approx 40.5$, $c \approx 9.8$**

11. $B = 47°$, $C = 112°$, $b = 13$ **$A = 21°$, $a \approx 6.4$, $c \approx 16.5$**

12. $A = 72°$, $a = 8$, $c = 6$ **$B \approx 62°$, $C \approx 46°$, $b \approx 7.5$**

13. $A = 25°$, $C = 107°$, $b = 12$ **$B = 48°$, $a \approx 6.8$, $c \approx 15.4$**

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

14. $A = 29°$, $a = 6$, $b = 13$ **no solution**

15. $A = 70°$, $a = 25$, $b = 20$ **one solution; $B \approx 49°$, $C \approx 61°$, $c \approx 23.3$**

16. $A = 113°$, $a = 21$, $b = 25$ **no solution**

17. $A = 110°$, $a = 20$, $b = 8$ **one solution; $B \approx 22°$, $C \approx 48°$, $c \approx 15.8$**

18. $A = 66°$, $a = 12$, $b = 7$ **one solution; $B \approx 32°$, $C \approx 82°$, $c \approx 13.0$**

19. $A = 54°$, $a = 5$, $b = 8$ **no solution**

20. $A = 45°$, $a = 15$, $b = 18$ **two solutions; $B \approx 58°$, $C \approx 77°$, $c \approx 20.7$; $B \approx 122°$, $C \approx 13°$, $c \approx 4.8$**

21. $A = 60°$, $a = 4\sqrt{3}$, $b = 8$ **one solution; $B = 90°$, $C = 30°$, $c \approx 4$**

22. WILDLIFE Sarah Phillips, an officer for the Department of Fisheries and Wildlife, checks boaters on a lake to make sure they do not disturb two osprey nesting sites. She leaves a dock and heads due north in her boat to the first nesting site. From here, she turns 5° north of due west and travels an additional 2.14 miles to the second nesting site. She then travels 6.7 miles directly back to the dock. How far from the dock is the first osprey nesting site? Round to the nearest tenth. **6.2 mi**

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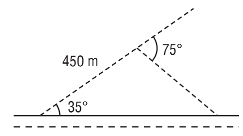
13-4 Word Problem Practice

Law of Sines

WALKING For Exercises 1 and 2, use the following information.

Alliya is taking a walk along a straight road. She decides to leave the road, so she walks on a path that makes an angle of 35° with the road. After walking for 450 meters, she turns 75° and heads back towards the road.

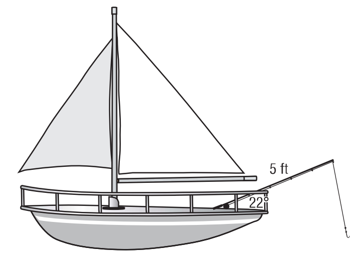


1. How far does Alliya need to walk on her current path to get back to the road? **402 m**

2. When Alliya returns to the road, how far along the road is she from where she started? **676 m**

3. ROCK CLIMBING A rock climber is part of the way up a climb when he can see both the peak and the base of the Gray Mountain. When viewing the peak of the mountain, his angle of elevation is 42°. When viewing the base of the mountain, his angle of depression is 36°. If he knows the Gray Mountain is 2000 feet high and the base of the mountain is at sea level, then what is the elevation of the climber to the nearest foot? **893 ft**

4. FISHING A fishing pole is resting against the railing of a boat making an angle of 22° with the boat's deck. The fishing pole is 5 feet long, and the hook hangs 3 feet from the tip of the pole. The movement of the boat causes the hook to sway back and forth. Determine which angles the fishing line must make with the pole in order for the hook to be level with the boat's deck. **119.4° or 16.6°**



CAMERAS For Exercises 5 and 6, use the following information.

A security camera is located on top of a building at a certain distance from the sidewalk. The camera revolves counterclockwise at a steady rate of one revolution per minute. At one point in the revolution it directly faces a point on the sidewalk that is 20 meters from the camera. 4 seconds later, it directly faces a point 10 meters down the sidewalk.

5. How many degrees does the camera rotate in 4 seconds? **24°**

6. To the nearest tenth of a meter, how far is the security camera from the sidewalk? **19.6 m**

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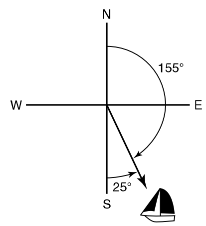
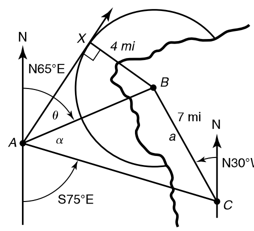
13-4 Enrichment

Navigation

The bearing of a boat is an angle showing the direction the boat is heading. Often, the angle is measured from north, but it can be measured from any of the four compass directions. At the right, the bearing of the boat is 155°. Or, it can be described as 25° east of south (S25°E).



Example **A boat A sights the lighthouse B in the direction N65°E and the spire of a church C in the direction S75°E. According to the map, B is 7 miles from C in the direction N30°W. In order for A to avoid running aground, find the bearing it should keep to pass B at 4 miles distance.**



In $\triangle ABC$, $\angle\alpha = 180° - 65° - 75°$ or $40°$

$\angle C = 180° - 30° - (180° - 75°)$
$= 45°$
$a = 7$ miles

With the Law of Sines,

$$AB = \frac{a \sin C}{\sin \alpha} = \frac{7(\sin 45°)}{\sin 40°} = 7.7 \text{ mi.}$$

The ray for the correct bearing for A must be tangent at X to circle B with radius $BX = 4$. Thus $\triangle ABX$ is a right triangle.

Then $\sin\theta = \frac{BX}{AB} = \frac{4}{7.7} \approx 0.519$. Therefore, $\angle\theta = 31°18'$.

The bearing of A should be $65° - 31°18'$ or $33°42'$.

Solve the following.

1. Suppose the lighthouse B in the example is sighted at S30°W by a ship P due north of the church C. Find the bearing P should keep to pass B at 4 miles distance. **S64°51′ W**

2. In the fog, the lighthouse keeper determines by radar that a boat 18 miles away is heading to the shore. The direction of the boat from the lighthouse is S80°E. What bearing should the lighthouse keeper radio the boat to take to come ashore 4 miles south of the lighthouse? **S87.2°E**

3. To avoid a rocky area along a shoreline, a ship at M travels 7 km to R, bearing 22°15′, then 8 km to P, bearing 68°30′, then 6 km to Q, bearing 109°15′. Find the distance from M to Q. **17.4 km**

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13-5 Lesson Reading Guide

Law of Cosines

Pre-Activity **How can you determine the angle at which to install a satellite dish?**

Read the introduction to Lesson 13-5 in your textbook.

One side of the triangle in the figure is not labeled with a length. What does the length of this side represent? Is this length greater than or less than the distance from the satellite to the equator?

the distance from the satellite to Valparaiso; greater than

Reading the Lesson

1. Each of the following equations can be changed into a correct statement of the Law of Cosines by making one change. In each case, indicate what change should be made to make the statement correct.

   a. $b^2 = a^2 + c^2 + 2ac \cos B$ — **Change the second + to −.**

   b. $a^2 = b^2 + c^2 - 2bc \sin A$ — **Change sin *A* to cos *A*.**

   c. $c = a^2 + b^2 - 2ab \cos C$ — **Change *c* to c^2.**

   d. $a^2 = b^2 - c^2 - 2bc \cos A$ — **Change the first − to +.**

2. Suppose that you are asked to solve $\triangle ABC$ given the following information about the sides and angles of the triangle. In each case, indicate whether you would begin by using the *Law of Sines* or the *Law of Cosines*.

   a. $a = 8, b = 7, c = 6$ — **Law of Cosines**

   b. $b = 9.5, A = 72°, B = 39°$ — **Law of Sines**

   c. $C = 123°, b = 22.95, a = 34.35$ — **Law of Cosines**

Helping You Remember

3. It is often easier to remember a complicated procedure if you can break it down into small steps. Describe in your own words how to use the Law of Cosines to find the length of one side of a triangle if you know the lengths of the other two sides and the measure of the included angle. Use numbered steps. (You may use mathematical terms, but do not use any mathematical symbols.)

   Sample answer: 1. Square each of the lengths of the two known sides. 2. Add these squares. 3. Find the cosine of the included angle. 4. Multiply this cosine by two times the product of the lengths of the two known sides. 5. Subtract the product from the sum. 6. Take the positive square root of the result.

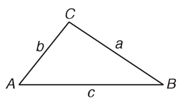
Lesson 13-5

NAME ______________________ DATE __________ PERIOD _____

13-5 Study Guide and Intervention

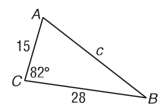
Law of Cosines

Law of Cosines

Law of Cosines	
	Let △*ABC* be any triangle with *a*, *b*, and *c* representing the measures of the sides, and opposite angles with measures *A*, *B*, and *C*, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

You can use the Law of Cosines to solve any triangle if you know the measures of two sides and the included angle, or the measures of three sides.

Example **Solve △*ABC*.**



You are given the measures of two sides and the included angle. Begin by using the Law of Cosines to determine *c*.

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 28^2 + 15^2 - 2(28)(15)\cos 82°$

$c^2 \approx 892.09$

$c \approx 29.9$

Next you can use the Law of Sines to find the measure of angle *A*.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{28} \approx \frac{\sin 82°}{29.9}$

$\sin A \approx 0.9273$

$A \approx 68°$

The measure of *B* is about $180° - (82° + 68°)$ or about 30°.

Exercises

Solve each triangle described below. Round measures of sides to the nearest tenth and angles to the nearest degree.

1. $a = 14, c = 20, B = 38°$
b ≈ 12.4, A ≈ 44°, C ≈ 98°

2. $A = 60°, c = 17, b = 12$
a ≈ 15.1, B ≈ 43°, C ≈ 77°

3. $a = 4, b = 6, c = 3$
A ≈ 36°, B ≈ 118°, C ≈ 26°

4. $A = 103°, b = 31, c = 52$
a ≈ 66, B ≈ 27°, C ≈ 50°

5. $a = 15, b = 26, C = 132°$
c ≈ 38, A ≈ 17°, B ≈ 31°

6. $a = 31, b = 52, c = 43$
A ≈ 36°, B ≈ 88°, C ≈ 56°

NAME ______________________ DATE __________ PERIOD _____

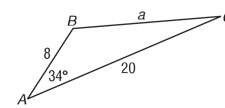
13-5 Study Guide and Intervention

Law of Cosines

Choose the Method

Solving an Oblique Triangle	Given	Begin by Using
	two angles and any side	Law of Sines
	two sides and a non-included angle	Law of Sines
	two sides and their included angle	Law of Cosines
	three sides	Law of Cosines

Example **Determine whether △*ABC* should be solved by beginning with the Law of Sines or Law of Cosines. Then solve the triangle. Round the measure of the side to the nearest tenth and measures of angles to the nearest degree.**



You are given the measures of two sides and their included angle, so use the Law of Cosines.

$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$a^2 = 20^2 + 8^2 - 2(20)(8) \cos 34°$ — $b = 20, c = 8, A = 34°$

$a^2 \approx 198.71$ — Use a calculator.

$a \approx 14.1$ — Use a calculator.

Use the Law of Sines to find *B*.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ — Law of Sines

$\sin B \approx \frac{20 \sin 34°}{14.1}$ — $b = 20, A = 34°, a \approx 14.1$

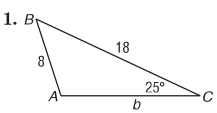
$B \approx 128°$ — Use the $\sin^{-1}$ function.

The measure of angle *C* is approximately $180° - (34° + 128°)$ or about 18°.

Exercises

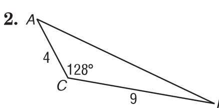
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1.



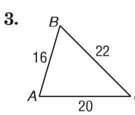
Law of Sines; A ≈ 108°, B ≈ 47°, b ≈ 13.8

2.



Law of Cosines; c ≈ 11.9, B ≈ 15°, A ≈ 37°

3.



Law of Cosines; A ≈ 74°, B ≈ 61°, C ≈ 45°

4. $A = 58°, a = 12, b = 8.5$
Law of Sines; B ≈ 37°, C ≈ 85°, c ≈ 14.1

5. $a = 28, b = 35, c = 20$
Law of Cosines; A ≈ 53°, B ≈ 92°, C ≈ 35°

6. $A = 82°, B = 44°, b = 11$
Law of Sines; a ≈ 15.7, c ≈ 12.8, C ≈ 54°

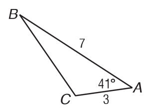
Lesson 13-5

NAME ______________________ DATE __________ PERIOD _____

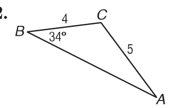
13-5 Skills Practice

Law of Cosines

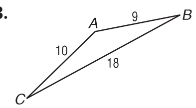
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. 

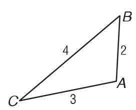
cosines; $B \approx 23°$, $C \approx 116°$, $a \approx 5.1$

2. 

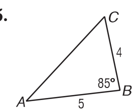
sines; $A \approx 27°$, $C \approx 119°$, $c \approx 7.9$

3. 

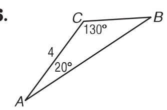
cosines; $A \approx 143°$, $B \approx 20°$, $C \approx 18°$

4. 

cosines; $A \approx 104°$, $B \approx 47°$, $C \approx 29°$

5. 

cosines; $A \approx 41°$, $C \approx 54°$, $b \approx 6.1$

6. 

sines; $B = 30°$, $a \approx 2.7$, $c \approx 6.1$

7. $C = 71°, a = 3, b = 4$
cosines; $A \approx 43°$, $B \approx 66°$, $c \approx 4.1$

8. $A = 11°, C = 27°, c = 50$
sines; $B = 142°$, $a \approx 21.0$, $b \approx 67.8$

9. $C = 35°, a = 5, b = 8$
cosines; $A \approx 37°$, $B \approx 108°$, $c \approx 4.8$

10. $B = 47°, a = 20, c = 24$
cosines; $A = 55°$, $C = 78°$, $b \approx 17.9$

11. $A = 71°, C = 62°, a = 20$
sines; $B = 47°$, $b \approx 15.5$, $c \approx 18.7$

12. $a = 5, b = 12, c = 13$
cosines; $A \approx 23°$, $B \approx 67°$, $C = 90°$

13. $A = 51°, b = 7, c = 10$
cosines; $B \approx 44°$, $C \approx 85°$, $a \approx 7.8$

14. $a = 13, A = 41°, B = 75°$
sines; $C = 64°$, $b \approx 19.1$, $c = 17.8$

15. $B = 125°, a = 8, b = 14$
sines; $A \approx 28°$, $C \approx 27°$, $c \approx 7.8$

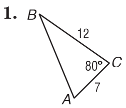
16. $a = 5, b = 6, c = 7$
cosines; $A \approx 44°$, $B \approx 57°$, $C \approx 78°$

NAME ______________________ DATE __________ PERIOD _____

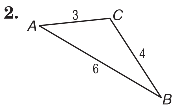
13-5 Practice (Average)

Law of Cosines

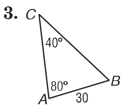
Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. 

cosines; $c \approx 12.8$, $A \approx 67°$, $B \approx 33°$

2. 

cosines; $A \approx 36°$, $B \approx 26°$, $C \approx 117°$

3. 

sines; $B = 60°$, $a \approx 46.0$, $b \approx 40.4$

4. $a = 16, b = 20, C = 54°$
cosines; $A \approx 51°$, $B \approx 75°$, $c \approx 16.7$

5. $B = 71°, c = 6, a = 11$
cosines; $A \approx 77°$, $C \approx 32°$, $b \approx 10.7$

6. $A = 37°, a = 20, b = 18$
sines; $B \approx 33°$, $C \approx 110°$, $c \approx 31.2$

7. $C = 35°, a = 18, b = 24$
cosines; $A \approx 48°$, $B \approx 97°$, $c \approx 13.9$

8. $a = 8, b = 6, c = 9$
cosines; $A \approx 61°$, $B \approx 41°$, $C \approx 79°$

9. $A = 23°, b = 10, c = 12$
cosines; $B \approx 54°$, $C \approx 103°$, $a \approx 4.8$

10. $a = 4, b = 5, c = 8$
cosines; $A \approx 24°$, $B \approx 31°$, $C \approx 125°$

11. $B = 46.6°, C = 112°, b = 13$
sines; $A = 21°$, $a \approx 6.5$, $c \approx 16.6$

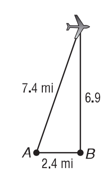
12. $A = 46.3°, a = 35, b = 30$
sines; $B \approx 38°$, $C \approx 95°$, $c \approx 48.2$

13. $a = 16.4, b = 21.1, c = 18.5$
cosines; $A \approx 48°$, $B \approx 74°$, $C \approx 57°$

14. $C = 43.5°, b = 8, c = 6$
sines; $A \approx 70°$, $B \approx 67°$, $a \approx 8.2$

15. $A = 78.3°, b = 7, c = 11$
cosines; $B \approx 36°$, $C \approx 66°$, $a \approx 11.8$

16. SATELLITES Two radar stations 2.4 miles apart are tracking an airplane. The straight-line distance between Station A and the plane is 7.4 miles. The straight-line distance between Station B and the plane is 6.9 miles. What is the angle of elevation from Station A to the plane? Round to the nearest degree. **69°**



17. DRAFTING Marion is using a computer-aided drafting program to produce a drawing for a client. She begins a triangle by drawing a segment 4.2 inches long from point A to point B. From B, she moves 42° degrees counterclockwise from the segment connecting A and B and draws a second segment that is 6.4 inches long, ending at point C. To the nearest tenth, how long is the segment from C to A? **9.9 in.**

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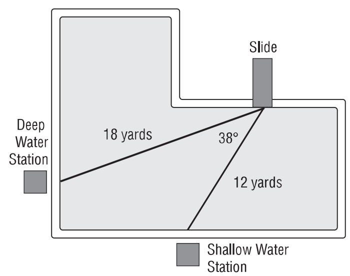
13-5 Word Problem Practice

Law of Cosines

POOLS For Exercises 1 and 2, use the following information.

The Perth County pool has a lifeguard station in both the deep water and shallow water sections of the pool. The distance between each station and the bottom of the slide is known, but the manager would like to calculate more information about the pool setup.



1. When the lifeguards switch positions, the lifeguard at the deep water station swims to the shallow water station. How far does the lifeguard swim? **13.8 yards**

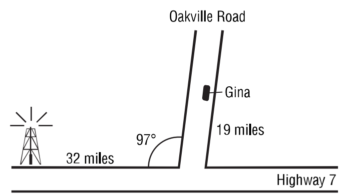
2. If the lifeguard at the deepwater station is directly facing the bottom of the slide, what angle does she need to turn in order to face the lifeguard at the shallow water station? **41.8°**

3. **CAMPING** At Shady Pines Campground, Campsites A and B are situated 80 meters apart. The camp office is 95 meters from Campsite A and 115 meters from Campsite B. When the ranger is standing at the office, what is the angle of separation between Campsites A and B? **43.5°**

4. **SKATING** During a figure skating routine, Jackie and Peter skate apart with an angle of 15° between them. Jackie skates for 5 meters and Peter skates for 7 meters. How far apart are the skaters? **2.5 m**

TECHNOLOGY For Exercises 5 and 6, use the following information.

Gina's handheld computer can send and receive e-mails if it is within 40 miles of a transmission tower. On a trip, Gina drives past the transmission tower on Highway 7 for 32 miles, and then she turns onto Oakville Road and drives for another 19 miles.



5. Is Gina close enough to the transmission tower to be able to send and receive e-mails? Explain your reasoning. **Yes. She is 39.2 miles from the tower.**

6. If Gina is within range of the tower, how much further can she drive on Oakville Road before she is out of range? If she is out of range and drives back towards Highway 7, how far will she travel before she is back in range? **She is within range and can drive 1.4 miles further.**

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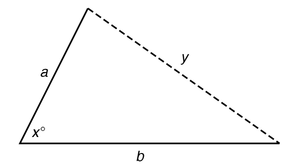
13-5 Enrichment

The Law of Cosines and the Pythagorean Theorem

The law of cosines bears strong similarities to the Pythagorean theorem. According to the law of cosines, if two sides of a triangle have lengths a and b and if the angle between them has a measure of $x°$, then the length, y, of the third side of the triangle can be found by using the equation



$y^2 = a^2 + b^2 - 2ab \cos x°$.

Answer the following questions to clarify the relationship between the law of cosines and the Pythagorean theorem.

1. If the value of $x°$ becomes less and less, what number is $\cos x°$ close to? **1**

2. If the value of $x°$ is very close to zero but then increases, what happens to $\cos x°$ as $x°$ approaches 90°? **decreases, approaches 0**

3. If $x°$ equals 90°, what is the value of $\cos x°$? What does the equation of $y^2 = a^2 + b^2 - 2ab \cos x°$ simplify to if $x°$ equals 90°? **0, $y^2 = a^2 + b^2$**

4. What happens to the value of $\cos x°$ as $x°$ increases beyond 90° and approaches 180°? **decreases to −1**

5. Consider some particular value of a and b, say 7 for a and 19 for b. Use a graphing calculator to graph the equation you get by solving $y^2 = 7^2 + 19^2 - 2(7)(19) \cos x°$ for y. **See students' graphs.**

   a. In view of the geometry of the situation, what range of values should you use for X? **X min = 0°; X max = 180°**

   b. Display the graph and use the TRACE function. What do the maximum and minimum values appear to be for the function? **See students' graphs.**

   c. How do the answers for part **b** relate to the lengths 7 and 19? Are the maximum and minimum values from part **b** ever actually attained in the geometric situation? **min = 19 − 7; max = 19 + 7; no**

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13-6 Lesson Reading Guide

Circular Functions

Pre-Activity **How can you model annual temperature fluctuations?**

Read the introduction to Lesson 13-6 in your textbook.

- If the graph in your textbook is continued, what month will $x = 17$ represent? **May of the following year**
- About what do you expect the average high temperature to be for that month? **24.2°F**
- Will this be exactly the average high temperature for that month? Explain your answer. **Sample answer: No; temperatures vary from year to year.**

Reading the Lesson

1. Use the unit circle on page 740 in your textbook to find the exact values of each expression.

a. $\cos 45°$ $\frac{\sqrt{2}}{2}$ **b.** $\sin 150°$ $\frac{1}{2}$ **c.** $\sin 240°$ $-\frac{\sqrt{3}}{2}$

d. $\sin 315°$ $-\frac{\sqrt{2}}{2}$ **e.** $\cos 270°$ **0** **f.** $\sin 210°$ $-\frac{1}{2}$

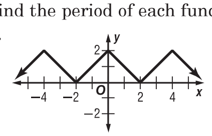
g. $\cos 0°$ **1** **h.** $\sin 180°$ **0** **i.** $\cos 330°$ $\frac{\sqrt{3}}{2}$

2. Tell whether each function is periodic. Write *yes* or *no*.

a. $y = 2x$ **no** **b.** $y = x^2$ **no** **c.** $y = \cos x$ **yes** **d.** $y = |x|$ **no**

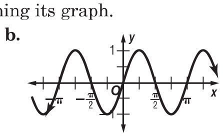
3. Find the period of each function by examining its graph.

a.



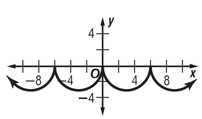
4

b.



π

c.



6

Helping You Remember

4. What is an easy way to remember the periods of the sine and cosine functions in radian measure? **Sample answer: The period of both functions is 2π, which is the circumference of the unit circle.**

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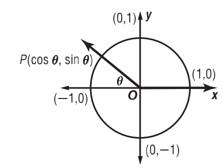
13-6 Study Guide and Intervention

Circular Functions

Unit Circle Definitions

Definition of Sine and Cosine	If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$.



Example **Given an angle θ in standard position, if $P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$ lies on the terminal side and on the unit circle, find $\sin \theta$ and $\cos \theta$.**

$P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right) = P(\cos \theta, \sin \theta)$, so $\sin \theta = \frac{\sqrt{11}}{6}$ and $\cos \theta = -\frac{5}{6}$.

Exercises

If θ is an angle in standard position and if the given point P is located on the terminal side of θ and on the unit circle, find $\sin \theta$ and $\cos \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\sin \theta = \frac{1}{2}$, $\cos \theta = -\frac{\sqrt{3}}{2}$

2. $P(0, -1)$
$\sin \theta = -1$, $\cos \theta = 0$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
$\sin \theta = \frac{\sqrt{5}}{3}$, $\cos \theta = -\frac{2}{3}$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$
$\sin \theta = -\frac{3}{5}$, $\cos \theta = -\frac{4}{5}$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$
$\sin \theta = -\frac{\sqrt{35}}{6}$, $\cos \theta = \frac{1}{6}$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$
$\sin \theta = \frac{3}{4}$, $\cos \theta = \frac{\sqrt{7}}{4}$

7. P is on the terminal side of $\theta = 45°$.
$\sin \theta = \frac{\sqrt{2}}{2}$, $\cos \theta = \frac{\sqrt{2}}{2}$

8. P is on the terminal side of $\theta = 120°$.
$\sin \theta = \frac{\sqrt{3}}{2}$, $\cos \theta = -\frac{1}{2}$

9. P is on the terminal side of $\theta = 240°$.
$\sin \theta = -\frac{\sqrt{3}}{2}$, $\cos \theta = -\frac{1}{2}$

10. P is on the terminal side of $\theta = 330°$.
$\sin \theta = -\frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$

NAME ______________ DATE ______ PERIOD ____

13-6 Study Guide and Intervention *(continued)*

Circular Functions

Periodic Functions

Periodic Functions	A function is called **periodic** if there is a number a such that $f(x) = f(x + a)$ for all x in the domain of the function. The least positive value of a for which $f(x) = f(x + a)$ is called the period of the function.

The sine and cosine functions are periodic; each has a period of 360° or 2π.

Example 1 **Find the exact value of each function.**

a. sin 855°

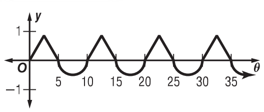
$\sin 855° = \sin(135° + 720°) = \sin 135° = \frac{\sqrt{2}}{2}$

b. $\cos\left(\frac{31\pi}{6}\right)$

$\cos\left(\frac{31\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + 4\pi\right)$

$= \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

Example 2 **Determine the period of the function graphed below.**



The pattern of the function repeats every 10 units, so the period of the function is 10.

Exercises

Find the exact value of each function.

1. cos (−240°) $-\frac{1}{2}$

2. cos 2880° **1**

3. sin (−510°) $-\frac{1}{2}$

4. sin 495° $\frac{\sqrt{2}}{2}$

5. $\cos\left(-\frac{5\pi}{2}\right)$ **0**

6. $\sin\left(\frac{5\pi}{3}\right)$ $-\frac{\sqrt{3}}{2}$

7. $\cos\left(\frac{11\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

8. $\sin\left(-\frac{3\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

9. cos 1440° **1**

10. sin (−750°) $-\frac{1}{2}$

11. cos 870° $-\frac{\sqrt{3}}{2}$

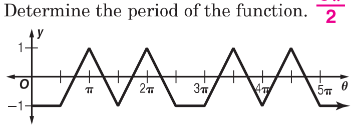
12. cos 1980° **−1**

13. sin 7π **0**

14. $\sin\left(-\frac{13\pi}{4}\right)$ $\frac{\sqrt{2}}{2}$

15. $\cos\left(\frac{23\pi}{6}\right)$ $\frac{\sqrt{3}}{2}$

16. Determine the period of the function. $\frac{5\pi}{2}$



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13-6 Study Guide and Intervention

Circular Functions

Lesson 13-6

The given point P is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.

1. $P\left(\frac{3}{5}, \frac{4}{5}\right)$ $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

2. $P\left(\frac{5}{13}, -\frac{12}{13}\right)$ $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$

3. $P\left(-\frac{9}{41}, -\frac{40}{41}\right)$ $\sin\theta = -\frac{40}{41}$, $\cos\theta = -\frac{9}{41}$

4. $P(0, 1)$ $\sin\theta = 1$, $\cos\theta = 0$

5. $P(-1, 0)$ $\sin\theta = 0$, $\cos\theta = -1$

6. $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\sin\theta = -\frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$

Find the exact value of each function.

7. cos 45° $\frac{\sqrt{2}}{2}$

8. sin 210° $-\frac{1}{2}$

9. sin 330° $-\frac{1}{2}$

10. cos 330° $\frac{\sqrt{3}}{2}$

11. cos (−60°) $\frac{1}{2}$

12. sin (−390°) $-\frac{1}{2}$

13. sin 5π **0**

14. cos 3π **−1**

15. $\sin\frac{5\pi}{2}$ **1**

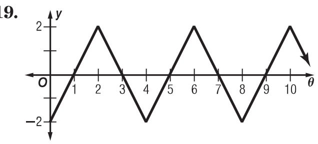
16. $\sin\frac{7\pi}{3}$ $\frac{\sqrt{3}}{2}$

17. $\cos\left(-\frac{7\pi}{3}\right)$ $\frac{1}{2}$

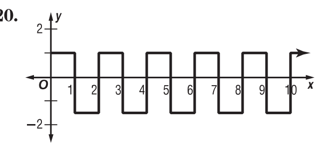
18. $\cos\left(-\frac{5\pi}{6}\right)$ $-\frac{\sqrt{3}}{2}$

Determine the period of each function.

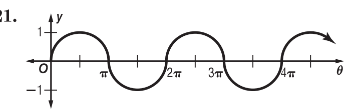
19. **4**



20. **2**



21. 2π



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13-6 Practice

Circular Functions

The given point P is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.

1. $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **$\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = -\frac{1}{2}$**

2. $P\left(\frac{20}{29}, -\frac{21}{29}\right)$ **$\sin\theta = -\frac{21}{29}$, $\cos\theta = \frac{20}{29}$**

3. $P(0.8, 0.6)$ **$\sin\theta = 0.6$, $\cos\theta = 0.8$**

4. $P(0, -1)$ **$\sin\theta = -1$, $\cos\theta = 0$**

5. $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ **$\sin\theta = -\frac{\sqrt{2}}{2}$, $\cos\theta = -\frac{\sqrt{2}}{2}$**

6. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **$\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$**

Find the exact value of each function.

7. $\cos\frac{7\pi}{4}$ **$\frac{\sqrt{2}}{2}$**

8. $\sin(-30°)$ **$-\frac{1}{2}$**

9. $\sin\left(-\frac{2\pi}{3}\right)$ **$-\frac{\sqrt{3}}{2}$**

10. $\cos(-330°)$ **$\frac{\sqrt{3}}{2}$**

11. $\cos 600°$ **$-\frac{1}{2}$**

12. $\sin\frac{9\pi}{2}$ **1**

13. $\cos 7\pi$ **-1**

14. $\cos\left(-\frac{11\pi}{4}\right)$ **$-\frac{\sqrt{2}}{2}$**

15. $\sin(-225°)$ **$\frac{\sqrt{2}}{2}$**

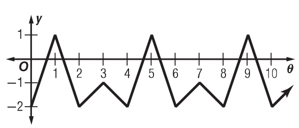
16. $\sin 585°$ **$-\frac{\sqrt{2}}{2}$**

17. $\cos\left(-\frac{10\pi}{3}\right)$ **$-\frac{1}{2}$**

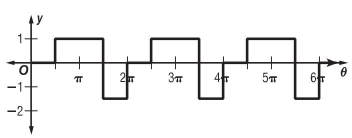
18. $\sin 840°$ **$\frac{\sqrt{3}}{2}$**

Determine the period of each function.

19. **4**



20. **2π**



21. FERRIS WHEELS A Ferris wheel with a diameter of 100 feet completes 2.5 revolutions per minute. What is the period of the function that describes the height of a seat on the outside edge of the Ferris Wheel as a function of time? **24 s**

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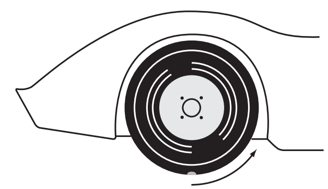
13-6 Study Guide and Intervention

Circular Functions

Lesson 13-6

Tires For Exercises 1–4, use the following information.

A point on the edge of a car tire is marked with paint. As the car moves slowly, the marked point on the tire varies in distance from the surface of the road. The height in inches of the point is given by the expression $h = -8\cos t + 8$, where t is the time in seconds.



1. What is the maximum height above ground that the point on the tire reaches? **16 inches**

2. What is the minimum height above ground that the point on the tire reaches? **0 inches**

3. How many rotations does the tire make per second? **1**

4. How far does the marked point travel in 30 seconds? How far does the marked point travel in one hour? **1508 ft; 2.9 mi**

Geometry For Exercises 5–8, use the following information.

The temperature T in degrees Fahrenheit of a city t months into the year is approximated by the formula $T = 42 + 30\sin\left(\frac{\pi}{6}t\right)$.

5. What is the highest monthly temperature for the city? **72°F**

6. In what month does the highest temperature occur? **March**

7. What is the lowest monthly temperature for the city? **12°F**

8. In what month does the lowest temperature occur? **September**

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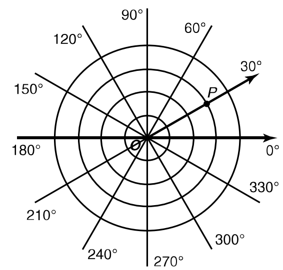
13-6 Enrichment

Polar Coordinates

Consider an angle in standard position with its vertex at a point O called the *pole*. Its initial side is on a coordinated axis called the *polar axis*. A point P on the terminal side of the angle is named by the *polar coordinates* (r, θ) where r is the directed distance of the point from O and θ is the measure of the angle.

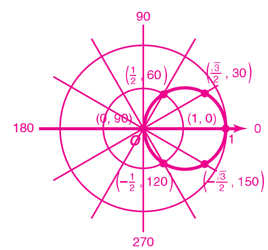
Graphs in this system may be drawn on polar coordinate paper such as the kind shown at the right.



The polar coordinates of a point are not unique. For example, $(3, 30°)$ names point P as well as $(3, 390°)$. Another name for P is $(-3, 210°)$. Can you see why? The coordinates of the pole are $(0, \theta)$ where θ may be any angle.

Example **Draw the graph of the function $r = \cos\theta$. Make a table of convenient values for θ and r. Then plot the points.**

θ	0°	30°	60°	90°	120°	150°	180°
r	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1



Since the period of the cosine function is 180°, values of r for $\theta > 180°$ are repeated.

Graph each function by making a table of values and plotting the values on polar coordinate paper.

1. $r = 4$
$r = 4$ for all values of θ. Graph should be a circle with radius 4 and center at the pole.

2. $r = 3 \sin\theta$
Graph is circle of radius $\frac{3}{2}$ with center at $\left(\frac{3}{2}, 90°\right)$.

3. $r = 3 \cos 2\theta$
Graph looks like flower with 4 petals, points of petals are at (3, 0°), (3, 90°), (3, 180°), (3, 270°). All petals meet at pole.

4. $r = 2(1 + \cos\theta)$
Graph is heart-shaped curve, symmetric with respect to polar axis.

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13-7 Lesson Reading Guide

Inverse Trigonometric Functions

Pre-Activity **How are inverse trigonometric functions used in road design?**

Read the introduction to Lesson 13-7 in your textbook.

Suppose you are given specific values for v and r. What feature of your graphing calculator could you use to find the approximate measure of the banking angle θ? **Sample answer: the TABLE feature**

Reading the Lesson

1. Indicate whether each statement is *true* or *false*.

a. The domain of the function $y = \sin x$ is the set of all real numbers. **true**

b. The domain of the function $y = \text{Cos } x$ is $0 \le x \le \pi$. **true**

c. The range of the function $y = \text{Tan } x$ is $-1 \le y \le 1$. **false**

d. The domain of the function $y = \text{Cos}^{-1} x$ is $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **false**

e. The domain of the function $y = \text{Tan}^{-1} x$ is the set of all real numbers. **true**

f. The range of the function $y = \text{Arcsin } x$ is $0 \le x \le \pi$. **false**

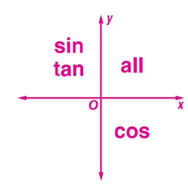
2. Answer each question in your own words.

a. What is the difference between the functions $y = \sin x$ and the function $y = \text{Sin } x$?
Sample answer: The domain of $y = \sin x$ is the set of all real numbers, while the domain of $y = \text{Sin } x$ is restricted to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

b. Why is it necessary to restrict the domains of the trigonometric functions in order to define their inverses? **Sample answer: Only one-to-one functions have inverses. None of the six basic trigonometric functions is one-to-one, but related one-to-one functions can be formed if the domains are restricted in certain ways.**

Helping You Remember

3. What is a good way to remember the domains of the functions $y = \text{Sin } x$, $y = \text{Cos } x$, and $y = \text{Tan } x$, which are also the range of the functions $y = \text{Arcsin } x$, $y = \text{Arccos } x$, and $y = \text{Arctan } x$? (You may want to draw a diagram.) **Sample answer: Each restricted domain must include an interval of numbers for which the function values are positive and one for which they are negative.**



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13-7 Study Guide and Intervention *(continued)*

Inverse Trigonometric Functions

Solve Equations Using Inverses If the domains of trigonometric functions are restricted to their **principal values**, then their inverses are also functions.

Principal Values of Sine, Cosine, and Tangent	$y = \text{Sin } x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. $y = \text{Cos } x$ if and only if $y = \cos x$ and $0 \le x \le \pi$. $y = \text{Tan } x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Inverse Sine, Cosine, and Tangent	Given $y = \text{Sin } x$, the inverse Sine function is defined by $y = \text{Sin}^{-1} x$ or $y = \text{Arcsin } x$. Given $y = \text{Cos } x$, the inverse Cosine function is defined by $y = \text{Cos}^{-1} x$ or $y = \text{Arccos } x$. Given $y = \text{Tan } x$, the inverse Tangent function is given by $y = \text{Tan}^{-1} x$ or $y = \text{Arctan } x$.

Example 1 **Solve $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$.**

If $x = \text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$, then $\text{Sin } x = \frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $x = \frac{\pi}{3}$ or 60°.

Example 2 **Solve $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right) = x$.**

If $x = \text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$, then $\text{Tan } x = -\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

The only x that satisfies both criteria is $-\frac{\pi}{6}$ or $-30°$.

Exercises

Solve each equation by finding the value of x to the nearest degree.

1. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) = x$ **150°**
2. $x = \text{Sin}^{-1}\frac{\sqrt{3}}{2}$ **60°**
3. $x = \text{Arccos}(-0.8)$ **143°**
4. $x = \text{Arctan}\sqrt{3}$ **60°**
5. $x = \text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$ **135°**
6. $x = \text{Tan}^{-1}(-1)$ **−45°**
7. $\text{Sin}^{-1} 0.45 = x$ **27°**
8. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$ **−60°**
9. $x = \text{Arccos}\left(-\frac{1}{2}\right)$ **120°**
10. $\text{Cos}^{-1}(-0.2) = x$ **102°**
11. $x = \text{Tan}^{-1}(-\sqrt{3})$ **−60°**
12. $x = \text{Arcsin } 0.3$ **17°**
13. $x = \text{Tan}^{-1}(15)$ **86°**
14. $x = \text{Cos}^{-1} 1$ **0**
15. $\text{Arctan}^{-1}(-3) = x$ **−72°**
16. $x = \text{Sin}^{-1}(-0.9)$ **−64°**
17. $\text{Arccos}^{-1} 0.15$ **81°**
18. $x = \text{Tan}^{-1} 0.2$ **11°**

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13-7 Study Guide and Intervention

Inverse Trigonometric Functions

Trigonometric Values You can use a calculator to find the values of trigonometric expressions.

Example **Find each value. Write angle measures in radians. Round to the nearest hundredth.**

a. Find $\tan\left(\text{Sin}^{-1}\frac{1}{2}\right)$.

Let $\theta = \text{Sin}^{-1}\frac{1}{2}$. Then $\text{Sin } \theta = \frac{1}{2}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The value $\theta = \frac{\pi}{6}$ satisfies both conditions. $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ so $\tan\left(\text{Sin}^{-1}\frac{1}{2}\right) = \frac{\sqrt{3}}{3}$.

b. Find $\cos(\text{Tan}^{-1} 4.2)$.

KEYSTROKES: COS 2nd [tan^{-1}] 4.2 ENTER .2316205273

Therefore $\cos(\text{Tan}^{-1} 4.2) \approx 0.23$.

Exercises

Find each value. Write angle measures in radians. Round to the nearest hundredth.

1. $\cot(\text{Tan}^{-1} 2)$ **0.5**
2. $\text{Arctan}(-1)$ **−0.79**
3. $\cot^{-1} 1$ **1.27**
4. $\cos\left[\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$ **0.71**
5. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **−1.05**
6. $\sin\left(\text{Arcsin}\frac{\sqrt{3}}{2}\right)$ **0.87**
7. $\tan\left[\text{Arcsin}\left(-\frac{5}{7}\right)\right]$ **−1.02**
8. $\sin\left(\text{Tan}^{-1}\frac{5}{12}\right)$ **0.38**
9. $\sin[\text{Arctan}^{-1}(-\sqrt{2})]$ **−0.82**
10. $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62**
11. $\text{Arcsin}\left(\frac{\sqrt{3}}{2}\right)$ **1.05**
12. $\text{Arccot}\left(-\frac{\sqrt{3}}{3}\right)$ **−1.91**
13. $\cos[\text{Arcsin}(-0.7)]$ **0.71**
14. $\tan(\text{Cos}^{-1} 0.28)$ **3.43**
15. $\cos(\text{Arctan } 5)$ **0.20**
16. $\text{Sin}^{-1}(-0.78)$ **−0.89**
17. $\text{Cos}^{-1} 0.42$ **1.14**
18. $\text{Arctan}(-0.42)$ **−0.40**
19. $\sin(\text{Cos}^{-1} 0.32)$ **0.95**
20. $\cos(\text{Arctan } 8)$ **0.12**
21. $\tan(\text{Cos}^{-1} 0.95)$ **0.33**

Lesson 13-7

Answers

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13-7 Skills Practice

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\alpha = \cos \beta$ **$\beta = \cos^{-1} \alpha$**
2. $\sin b = a$ **$\sin^{-1} a = b$**
3. $y = \tan x$ **$x = \tan^{-1} y$**
4. $\cos 45° = \frac{\sqrt{2}}{2}$ **$\cos^{-1} \frac{\sqrt{2}}{2} = 45°$**
5. $b = \sin 150°$ **$150° = \sin^{-1} b$**
6. $\tan y = \frac{4}{5}$ **$\tan^{-1} \frac{4}{5} = y$**

Solve each equation by finding the value of x to the nearest degree.

7. $x = \text{Cos}^{-1}(-1)$ **180°**
8. $\text{Sin}^{-1}(-1) = x$ **−90°**
9. $\text{Tan}^{-1} 1 = x$ **45°**
10. $x = \text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right)$ **−60°**
11. $x = \text{Arctan } 0$ **0°**
12. $x = \text{Arccos } \frac{1}{2}$ **60°**

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Sin}^{-1} \frac{\sqrt{2}}{2}$ **0.79 radians**
14. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62 radians**
15. $\text{Tan}^{-1} \sqrt{3}$ **1.05 radians**
16. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$ **−0.52 radians**
17. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$ **2.36 radians**
18. $\text{Arcsin } 1$ **1.57 radians**
19. $\sin(\text{Cos}^{-1} 1)$ **0**
20. $\sin\left(\text{Sin}^{-1} \frac{1}{2}\right)$ **0.5**
21. $\tan\left(\text{Arcsin} \frac{\sqrt{3}}{2}\right)$ **1.73**
22. $\cos(\text{Tan}^{-1} 3)$ **0.32**
23. $\sin[\text{Arctan}(-1)]$ **−0.71**
24. $\sin\left[\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)\right]$ **0.71**

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13-7 Practice (Average)

Inverse Trigonometric Functions

Write each equation in the form of an inverse function.

1. $\beta = \cos \alpha$ **$\alpha = \cos^{-1} \beta$**
2. $\tan \beta = \alpha$ **$\beta = \tan^{-1} \alpha$**
3. $y = \tan 120°$ **$120° = \tan^{-1} y$**
4. $-\frac{1}{2} = \cos x$ **$x = \cos^{-1}\left(-\frac{1}{2}\right)$**
5. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ **$\sin^{-1} \frac{\sqrt{3}}{2} = \frac{2\pi}{3}$**
6. $\cos \frac{\pi}{3} = \frac{1}{2}$ **$\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$**

Solve each equation by finding the value of x to the nearest degree.

7. $\text{Arcsin } 1 = x$ **90°**
8. $\text{Cos}^{-1} \frac{\sqrt{3}}{2} = x$ **30°**
9. $x = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ **−30°**
10. $x = \text{Arccos } \frac{\sqrt{2}}{2}$ **45°**
11. $x = \text{Arctan}(-\sqrt{3})$ **−60°**
12. $\text{Sin}^{-1}\left(-\frac{1}{2}\right) = x$ **−30°**

Find each value. Write angle measures in radians. Round to the nearest hundredth.

13. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **2.62 radians**
14. $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **−0.79 radians**
15. $\text{Arctan}\left(-\frac{\sqrt{3}}{3}\right)$ **−0.52 radians**
16. $\tan\left(\text{Cos}^{-1} \frac{1}{2}\right)$ **1.73**
17. $\cos\left[\text{Sin}^{-1}\left(-\frac{3}{5}\right)\right]$ **0.8**
18. $\cos[\text{Arctan}(-1)]$ **0.71**
19. $\tan\left(\sin^{-1} \frac{12}{13}\right)$ **2.4**
20. $\sin\left(\text{Arctan} \frac{\sqrt{3}}{3}\right)$ **0.5**
21. $\text{Cos}^{-1}\left(\tan \frac{3\pi}{4}\right)$ **3.14 radians**
22. $\text{Sin}^{-1}\left(\cos \frac{\pi}{3}\right)$ **0.52 radians**
23. $\sin\left(2\ \text{Cos}^{-1} \frac{15}{17}\right)$ **0.83**
24. $\cos\left(2\ \text{Sin}^{-1} \frac{\sqrt{3}}{2}\right)$ **−0.5**

25. **PULLEYS** The equation $x = \cos^{-1} 0.95$ describes the angle through which pulley A moves, and $y = \cos^{-1} 0.17$ describes the angle through which pulley B moves. Both angles are greater than 270° and less than 360°. Which pulley moves through a greater angle? **pulley *A***

26. **FLYWHEELS** The equation $y = \text{Arctan } 1$ describes the counterclockwise angle through which a flywheel rotates in 1 millisecond. Through how many degrees has the flywheel rotated after 25 milliseconds? **1125°**