NAME ______________________ DATE ____________ PERIOD _____

10-1 Skills Practice

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (4, −1), (−4, 1) **(0, 0)**

2. (−1, 4), (5, 2) **(2, 3)**

3. (3, 4), (5, 4) **(4, 4)**

4. (6, 2), (2, −1) $\left(4, \frac{1}{2}\right)$

5. (3, 9), (−2, −3) $\left(\frac{1}{2}, 3\right)$

6. (−3, 5), (−3, −8) $\left(-3, -\frac{3}{2}\right)$

7. (3, 2), (−5, 0) **(−1, 1)**

8. (3, −4), (5, 2) **(4, −1)**

9. (−5, −9), (5, 4) $\left(0, -\frac{5}{2}\right)$

10. (−11, 14), (0, 4) $\left(-\frac{11}{2}, 9\right)$

11. (3, −6), (−8, −3) $\left(-\frac{5}{2}, -\frac{9}{2}\right)$

12. (0, 10), (−2, −5) $\left(-1, \frac{5}{2}\right)$

Find the distance between each pair of points with the given coordinates.

13. (4, 12), (−1, 0) **13 units**

14. (7, 7), (−5, −2) **15 units**

15. (−1, 4), (1, 4) **2 units**

16. (11, 11), (8, 15) **5 units**

17. (1, −6), (7, 2) **10 units**

18. (3, −5), (3, 4) **9 units**

19. (2, 3), (3, 5) $\sqrt{5}$ **units**

20. (−4, 3), (−1, 7) **5 units**

21. (−5, −5), (3, 10) **17 units**

22. (3, 9), (−2, −3) **13 units**

23. (6, −2), (−1, 3) $\sqrt{74}$ **units**

24. (−4, 1), (2, −4) $\sqrt{61}$ **units**

25. (0, −3), (4, 1) $4\sqrt{2}$ **units**

26. (−5, −6), (2, 0) $\sqrt{85}$ **units**

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10-1 Practice

Midpoint and Distance Formulas

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (8, −3), (−6, −11) **(1, −7)**

2. (−14, 5), (10, 6) $\left(-2, \frac{11}{2}\right)$

3. (−7, −6), (1, −2) **(−3, −4)**

4. (8, −2), (8, −8) **(8, −5)**

5. (9, −4), (1, −1) $\left(5, -\frac{5}{2}\right)$

6. (3, 3), (4, 9) $\left(\frac{7}{2}, 6\right)$

7. (4, −2), (3, −7) $\left(\frac{7}{2}, -\frac{9}{2}\right)$

8. (6, 7), (4, 4) $\left(5, \frac{11}{2}\right)$

9. (−4, −2), (−8, 2) **(−6, 0)**

10. (5, −2), (3, 7) $\left(4, \frac{5}{2}\right)$

11. (−6, 3), (−5, −7) $\left(-\frac{11}{2}, -2\right)$

12. (−9, −8), (8, 3) $\left(-\frac{1}{2}, -\frac{5}{2}\right)$

13. (2.6, −4.7), (8.4, 2.5) **(5.5, −1.1)**

14. $\left(-\frac{1}{3}, 6\right), \left(\frac{2}{3}, 4\right)$ $\left(\frac{1}{6}, 5\right)$

15. (−2.5, −4.2), (8.1, 4.2) **(2.8, 0)**

16. $\left(\frac{1}{8}, \frac{1}{2}\right), \left(-\frac{5}{8}, -\frac{1}{2}\right)$ $\left(-\frac{1}{4}, 0\right)$

Find the distance between each pair of points with the given coordinates.

17. (5, 2), (2, −2) **5 units**

18. (−2, −4), (4, 4) **10 units**

19. (−3, 8), (−1, −5) $\sqrt{173}$ **units**

20. (0, 1), (9, −6) $\sqrt{130}$ **units**

21. (−5, 6), (−6, 6) **1 unit**

22. (−3, 5), (12, −3) **17 units**

23. (−2, −3), (9, 3) $\sqrt{157}$ **units**

24. (−9, −8), (−7, 8) $2\sqrt{65}$ **units**

25. (9, 3), (9, −2) **5 units**

26. (−1, −7), (0, 6) $\sqrt{170}$ **units**

27. (10, −3), (−2, −8) **13 units**

28. (−0.5, −6), (1.5, 0) $2\sqrt{10}$ **units**

29. $\left(\frac{2}{5}, \frac{3}{5}\right), \left(1, \frac{7}{5}\right)$ **1 unit**

30. $(-4\sqrt{2}, -\sqrt{5}), (-5\sqrt{2}, 4\sqrt{5})$ $\sqrt{127}$ **units**

31. GEOMETRY Circle O has a diameter $\overline{AB}$. If A is at (−6, −2) and B is at (−3, 4), find the center of the circle and the length of its diameter. $\left(-\frac{9}{2}, 1\right)$; $3\sqrt{5}$ **units**

32. GEOMETRY Find the perimeter of a triangle with vertices at (1, −3), (−4, 9), and (−2, 1). $18 + 2\sqrt{17}$ **units**

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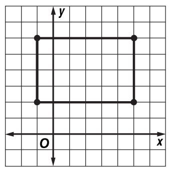
10-1 Word Problem Practice

Midpoint and Distance Formulas

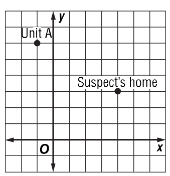
1. **EXHIBITS** Museum planners want to place a statue directly in the center of their Special Exhibits Room. Suppose the room is placed on a coordinate plane as shown. What are the coordinates of the center of this room?



(2, 4)

2. **WALKING** Laura starts at the origin. She walks 8 units to the right and then 12 units up. How far away from the origin is she? Round your answer to the nearest tenth.
14.4 units

3. **SURVEILLANCE** A grid is superimposed on a map of the area directly surrounding the home of a suspect. Detectives want to position themselves on opposite sides of the suspect's house. Coordinates are assigned to the suspect's home. Unit A is positioned at $(-1, 6)$ on the coordinate plane. Where should Unit B be located so that the suspect's home is centered between the two units?

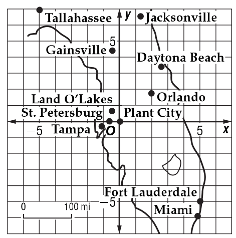


(9, 0)

4. **AIRPLANES** A grid is superimposed on a map of Texas. Dallas has coordinates (200, 5) and Amarillo has coordinates $(-100, 208)$. If each unit represents 1 mile, how long will it take a plane flying at an average speed of 410 miles per hour to fly directly from Dallas to Amarillo? Round your answer to the nearest tenth of an hour.
0.9 hour

TRAVEL For Exercises 5 and 6, use the following information and the figure below.

The Martinez family is planning a trip from their home in Fort Lauderdale to Tallahassee. They plan to stop overnight at a location about halfway between the two cities.



5. What are the coordinates of the point halfway between Tallahassee and Fort Lauderdale? Which of the cities on the map is closest to this point?
(0, 1); Land O'Lakes

6. How many miles is it from Fort Lauderdale to Tallahassee? Round your answer to the nearest mile.
521 mi

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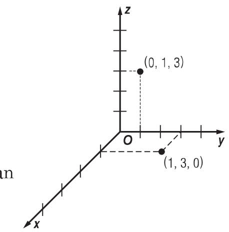
10-1 Enrichment

Distance Between Points in Space

The Distance Formula and Midpoint Formula on the coordinate plane is derived from the Pythagorean Theorem $a^2 + b^2 = c^2$.

In three dimensions, the coordinate grid contains the x-axis and the y-axis, as in two-dimensional geometry, and also a z-axis. An example of a line segment drawn on a three-dimensional coordinate grid is shown at the right.



The three-dimensional distance formula is much like the one for two dimensions. The distance from $A\ (x_1, y_1, z_1)$ to $B\ (x_2, y_2, z_2)$ can be found using $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$.

Example **Find the distance between the points (1, 3, 0) and (0, 1, 3).**

$d = \sqrt{(1 - 0)^2 + (3 - 1)^2 + (0 - 3)^2}$ Replace (x_1, y_1, z_1) with (1, 3, 0) and (x_2, y_2, z_2) with (0, 1, 3).

$d = \sqrt{1 + 4 + 9}$ or $\sqrt{14}$ Simplify.

Exercises

1. Find the distance between each pair of points A (1, 3, −2) and B (4, 2, 1).
$d = \sqrt{19}$

2. Find the distance between each pair of points C (5, 3, 2) and D (0, 1, 7).
$d = \sqrt{54}$

3. Find the distance between each pair of points E (−2, −1, 6) and F (−1, 3, 2).
$d = \sqrt{33}$

4. Use what you know about the midpoint formula for a segment graphed on a regular coordinate grid to make a conjecture about the formula for finding the coordinates of a midpoint in three-dimensions.
Midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

5. Find the midpoint for each segment in Exercises 1–3.
(5/2, 5/2, −1/2), (5/2, 2, 9/2), (−3/2, 1, 4)

Lesson 10-1

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10-2 Lesson Reading Guide

Parabolas

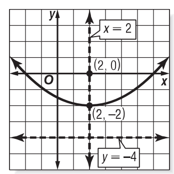
Get Ready for the Lesson

Read the introduction to Lesson 10-2 in your textbook.

Name at least two reflective objects that might have the shape of a parabola.

Sample answer: telescope mirror, satellite dish

Read the Lesson



1. In the parabola shown in the graph, the point $(2, -2)$ is called the **vertex** and the point $(2, 0)$ is called the **focus**. The line $y = -4$ is called the **directrix**, and the line $x = 2$ is called the **axis of symmetry**.

2. a. Write the standard form of the equation of a parabola that opens upward or downward. $y = a(x - h)^2 + k$

b. The parabola opens downward if $a < 0$ and opens upward if $a > 0$. The equation of the axis of symmetry is $x = h$, and the coordinates of the vertex are (h, k).

3. A parabola has equation $x = -\frac{1}{8}(y - 2)^2 + 4$. This parabola opens to the **left**. It has vertex $(4, 2)$ and focus $(2, 2)$. The directrix is $x = 6$. The length of the latus rectum is **8** units.

Remember What You Learned

4. How can the way in which you plot points in a rectangular coordinate system help you to remember what the sign of a tells you about the direction in which a parabola opens?

Sample answer: In plotting points, a positive *x*-coordinate tells you to move to the *right* and a negative *x*-coordinate tells you to move to the *left*. This is like a parabola whose equation is of the form "$x = \ldots$"; it opens to the *right* if $a > 0$ and to the *left* if $a < 0$. Likewise, a positive *y*-coordinate tells you to move *up* and a negative *y*-coordinate tells you to move *down*. This is like a parabola whose equation is of the form "$y = \ldots$"; it opens *upward* if $a > 0$ and *downward* if $a < 0$.

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10-2 Study Guide and Intervention

Parabolas

Equations of Parabolas A parabola is a curve consisting of all points in the coordinate plane that are the same distance from a given point (the **focus**) and a given line (the **directrix**). The following chart summarizes important information about parabolas.

Standard Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Axis of Symmetry	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\lvert\frac{1}{a}\right\rvert$ units	$\left\lvert\frac{1}{a}\right\rvert$ units

Example **Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = 2x^2 - 12x - 25$.**

$y = 2x^2 - 12x - 25$	Original equation
$y = 2(x^2 - 6x) - 25$	Factor 2 from the *x*-terms.
$y = 2(x^2 - 6x + ■) - 25 - 2(■)$	Complete the square on the right side.
$y = 2(x^2 - 6x + 9) - 25 - 2(9)$	The 9 added to complete the square is multiplied by 2.
$y = 2(x - 3)^2 - 43$	Write in standard form.

The vertex of this parabola is located at $(3, -43)$, the focus is located at $\left(3, -42\frac{7}{8}\right)$, the equation of the axis of symmetry is $x = 3$, and the equation of the directrix is $y = -43\frac{1}{8}$. The parabola opens upward.

Exercises

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation.

1. $y = x^2 + 6x - 4$

$(-3, -13)$, $\left(-3, -12\frac{3}{4}\right)$, $x = -3$, $y = -13\frac{1}{4}$, up

2. $y = 8x - 2x^2 + 10$

$(2, 18)$, $\left(2, 17\frac{1}{8}\right)$, $x = 2$, $y = 18\frac{1}{8}$, down

3. $x = y^2 - 8y + 6$

$(-10, 4)$, $\left(-9\frac{3}{4}, 4\right)$, $y = 4$, $x = -10\frac{1}{4}$, right

Write an equation of each parabola described below.

4. focus $(-2, 3)$, directrix $x = -2\frac{1}{12}$

$x = 6(y - 3)^2 - 2\frac{1}{24}$

5. vertex $(5, 1)$, focus $\left(4\frac{11}{12}, 1\right)$

$x = -3(y - 1)^2 + 5$

Lesson 10-2

Answers

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10-2 Study Guide and Intervention *(continued)*

Parabolas

Graph Parabolas To graph an equation for a parabola, first put the given equation in standard form.

$y = a(x - h)^2 + k$ for a parabola opening up or down, or
$x = a(y - k)^2 + h$ for a parabola opening to the left or right

Use the values of a, h, and k to determine the vertex, focus, axis of symmetry, and length of the latus rectum. The vertex and the endpoints of the latus rectum give three points on the parabola. If you need more points to plot an accurate graph, substitute values for points near the vertex.

Example **Graph $y = \frac{1}{3}(x - 1)^2 + 2$.**

In the equation, $a = \frac{1}{3}$, $h = 1$, $k = 2$.

The parabola opens up, since $a > 0$.

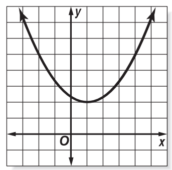
vertex: (1, 2)

axis of symmetry: $x = 1$

focus: $\left(1, 2 + \frac{1}{4\left(\frac{1}{3}\right)}\right)$ or $\left(1, 2\frac{3}{4}\right)$

length of latus rectum: $\left|\frac{1}{\frac{1}{3}}\right|$ or 3 units

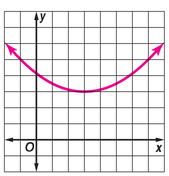
endpoints of latus rectum: $\left(2\frac{1}{2}, 2\frac{3}{4}\right), \left(-\frac{1}{2}, 2\frac{3}{4}\right)$



Exercises

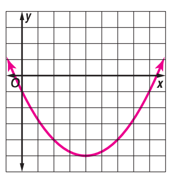
The coordinates of the focus and the equation of the directrix of a parabola are given. Write an equation for each parabola and draw its graph.

1. (3, 5), $y = 1$



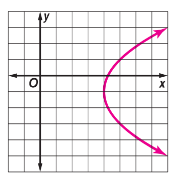
$y = \frac{1}{8}(x - 3)^2 + 3$

2. (4, −4), $y = -6$



$y = \frac{1}{4}(x - 4)^2 - 5$

3. (5, −1), $x = 3$



$x = \frac{1}{4}(y + 1)^2 + 4$

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10-2 Skills Practice

Parabolas

Write each equation in standard form.

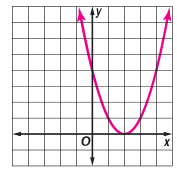
1. $y = x^2 + 2x + 2$
$y = [x - (-1)]^2 + 1$

2. $y = x^2 - 2x + 4$
$y = (x - 1)^2 + 3$

3. $y = x^2 + 4x + 1$
$y = [x - (-2)]^2 + (-3)$

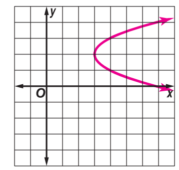
Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y = (x - 2)^2$



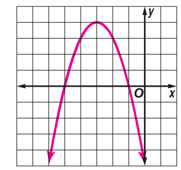
vertex: (2, 0);
focus: $\left(2, \frac{1}{4}\right)$;
axis of symmetry: $x = 2$;
directrix: $y = -\frac{1}{4}$;
opens up;
latus rectum: 1 unit

5. $x = (y - 2)^2 + 3$



vertex: (3, 2);
focus: $\left(3\frac{1}{4}, 2\right)$;
axis of symmetry: $y = 2$;
directrix: $x = 2\frac{3}{4}$;
opens right;
latus rectum: 1 unit

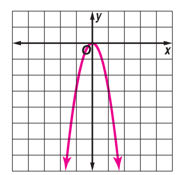
6. $y = -(x + 3)^2 + 4$



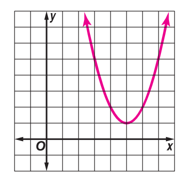
vertex: (−3, 4);
focus: $\left(-3, 3\frac{3}{4}\right)$;
axis of symmetry: $x = -3$;
directrix: $y = 4\frac{1}{4}$;
opens down;
latus rectum: 1 unit

Write an equation for each parabola described below. Then draw the graph.

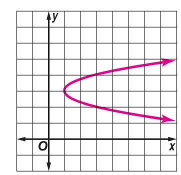
7. vertex (0, 0), focus $\left(0, -\frac{1}{12}\right)$
$y = -3x^2$



8. vertex (5, 1), focus $\left(5, \frac{5}{4}\right)$
$y = (x - 5)^2 + 1$



9. vertex (1, 3), directrix $x = \frac{7}{8}$
$x = 2(y - 3)^2 + 1$



NAME ______________________ DATE __________ PERIOD _____

10-2 Practice

Parabolas

Write each equation in standard form.

1. $y = 2x^2 - 12x + 19$

$y = 2(x - 3)^2 + 1$

2. $y = \frac{1}{2}x^2 + 3x + \frac{1}{2}$

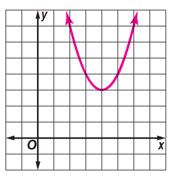
$y = \frac{1}{2}[x - (-3)]^2 + (-4)$

3. $y = -3x^2 - 12x - 7$

$y = -3[x - (-2)]^2 + 5$

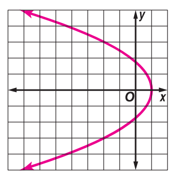
Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y = (x - 4)^2 + 3$



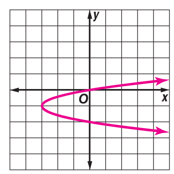
vertex: (4, 3);
focus: $\left(4, 3\frac{1}{4}\right)$;
axis: $x = 4$;
directrix: $y = 2\frac{3}{4}$;
opens up;
latus rectum: 1 unit

5. $x = -\frac{1}{3}y^2 + 1$



vertex: (1, 0);
focus: $\left(\frac{1}{4}, 0\right)$;
axis: $y = 0$;
directrix: $x = 1\frac{3}{4}$;
opens left;
latus rectum: 3 units

6. $x = 3(y + 1)^2 - 3$

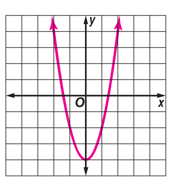


vertex: (−3, −1);
focus: $\left(-2\frac{11}{12}, -1\right)$;
axis: $y = -1$;
directrix: $x = -3\frac{1}{12}$;
opens right;
latus rectum: $\frac{1}{3}$ unit

Write an equation for each parabola described below. Then draw the graph.

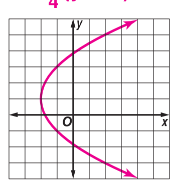
7. vertex $(0, -4)$, focus $\left(0, -3\frac{7}{8}\right)$

$y = 2x^2 - 4$



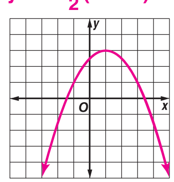
8. vertex $(-2, 1)$, directrix $x = -3$

$x = \frac{1}{4}(y - 1)^2 - 2$



9. vertex $(1, 3)$, axis of symmetry $x = 1$, latus rectum: 2 units, $a < 0$

$y = -\frac{1}{2}(x - 1)^2 + 3$



10. TELEVISION Write the equation in the form $y = ax^2$ for a satellite dish. Assume that the bottom of the upward-facing dish passes through (0, 0) and that the distance from the bottom to the focus point is 8 inches. **$y = \frac{1}{32}x^2$**

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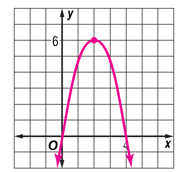
10-2 Word Problem Practice

Parabolas

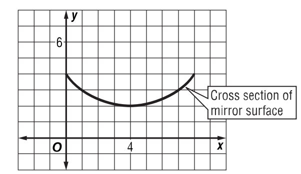
1. PROJECTILE A projectile follows the graph of the parabola $y = -\frac{3}{2}x^2 + 6x$. Sketch the path of the projectile by graphing the parabola.



2. COMMUNICATION David has just made a large parabolic dish whose cross section is based on the graph of the parabola $y = 0.25x^2$. Each unit represents one foot and the diameter of his dish is 4 feet. He wants to make a listening device by placing a microphone at the focus of the parabola. Where should the microphone be placed?
1 foot away from the vertex along the axis (at the point (0, 1) with respect to the graph)

3. BRIDGES A bridge is in the shape of a parabola that opens downward. The equation of the parabola to model the arch of the bridge is given by $y = -\frac{x^2}{24} + \frac{5}{6}x + \frac{11}{6}$, where each unit is equivalent to 1 yard. The x-axis is the ground level. What is the maximum height of the bridge above the ground?
6 yd

4. TELESCOPES An astronomer is working with a large reflecting telescope. The reflecting mirror in the telescope has the parabolic cross section shown in the graph whose equation is given by $y = \frac{1}{8}(x - 4)^2 + 2$. Each unit represents 1 meter. The astronomer is standing at the origin. How far from the focus of the parabola is the point on the mirror directly over the astronomer's head?



4 m

BRIDGES For Exercises 5 and 6, use the following information.
Part of the Sydney Harbor Bridge in Sydney, Australia, can be modeled by a parabolic arch. If each unit corresponds to 10 meters, the arch would pass through the points at (−25, 5), (0, 10), and (25, 5).

5. Write the equation of the parabola to model the arch.
$y = -\frac{1}{125}x^2 + 10$

6. Identify the coordinates of the focus of this parabola.
(0, −21.25)

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10-2 Enrichment

Limits

Sequences of numbers with a rational expression for the general term often approach some number as a finite limit. For example, the reciprocals of the positive integers approach 0 as n gets larger and larger. This is written using the notation shown below. The symbol ∞ stands for infinity and $n \to \infty$ means that n is getting larger and larger, or "n goes to infinity."

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$ $\qquad \lim_{n \to \infty} \frac{1}{n} = 0$

Example **Find $\lim_{n \to \infty} \frac{n^2}{(n+1)^2}$**

It is not immediately apparent whether the sequence approaches a limit or not. But notice what happens if we divide the numerator and denominator of the general term by n^2.

$$\frac{n^2}{(n+1)^2} = \frac{n^2}{n^2 + 2n + 1}$$

$$= \frac{\frac{n^2}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}$$

$$= \frac{1}{1 + \frac{2}{n} + \frac{1}{n^2}}$$

The two fractions in the denominator will approach a limit of 0 as n gets very large, so the entire expression approaches a limit of 1.

Exercises

Find the following limits.

1. $\lim_{n \to \infty} \frac{n^3 + 5n}{n^4 - 6}$ **0**

2. $\lim_{n \to \infty} \frac{1 - n}{n^2}$ **0**

3. $\lim_{n \to \infty} \frac{2(n+1) + 1}{2n + 1}$ **1**

4. $\lim_{n \to \infty} \frac{2n + 1}{1 - 3n}$ $-\frac{2}{3}$

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10-2 Spreadsheet Activity

Parabolas

You have learned many of the characteristics of parabolas with vertical and horizontal axes of symmetry. The information is summarized in the table at the right. You can use what you know to create a spreadsheet to analyze given equations of parabolas.

form of equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
vertex	(h, k)	(h, k)
axis of symmetry	$x = h$	$y = k$
focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
direction of opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
length of latus rectum	$\left\lvert\frac{1}{a}\right\rvert$ units	$\left\lvert\frac{1}{a}\right\rvert$ units

Exercises

The spreadsheet below uses the equation of a parabola in the form $y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$ to find information about the parabola. x or y is entered in Column D and the values of a, h, and k are entered into Columns A, B, and C respectively.

◇	A	B	C	D	E	F	G	H	I	J
1	a	h	k	y = or x =	Vertex	Length of Latus Rectum (units)	Axis of Symmetry	Focus	Directrix	Direction of Opening
2	0.25	3.00	−1	x	3, −1	4.00	y = −1	4, −1	x = 2	Right
3	3.00	−4.00	2.00	y	−4, 2	0.33	x = −4	−4, 2.08	y = 1.92	Upward
4	−1.00	3.00	2.00	y	3, 2	1.00	x = 3	3, 1.75	y = 2.25	Downward

Sheet 1 / Sheet 2 / Sheet 3

1. Which row represents the equation $y = 3x^2 + 24x + 50$? **row 3**

2. Write the standard form of the equation represented by row 2.
$x = \frac{1}{4}(y + 1)^2 + 3$

3. What formula should be used in cell F2? **1/ABS(A2)**

4. Find the vertex, length of latus rectum, axis of symmetry, focus, directrix, and direction of opening of a parabola with equation $(y - 8)^2 = -4(x - 4)$.
(8, 4); 4; $y = 4$; (7, 4); $x = 9$; left

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10-3 Lesson Reading Guide

Circles

Get Ready for the Lesson

Read the introduction to Lesson 10-3 in your textbook.

A large home improvement chain is planning to enter a new metropolitan area and needs to select locations for its stores. Market research has shown that potential customers are willing to travel up to 12 miles to shop at one of their stores. How can circles help the managers decide where to place their store?

Sample answer: A store will draw customers who live inside a circle with center at the store and a radius of 12 miles. The management should select locations for which as many people as possible live within a circle of radius 12 miles around one of the stores.

Read the Lesson

1. a. Write the equation of the circle with center (h, k) and radius r.
$(x - h)^2 + (y - k)^2 = r^2$

b. Write the equation of the circle with center $(4, -3)$ and radius 5.
$(x - 4)^2 + (y + 3)^2 = 25$

c. The circle with equation $(x + 8)^2 + y^2 = 121$ has center **(−8, 0)** and radius **11**.

d. The circle with equation $(x - 10)^2 + (y + 10)^2 = 1$ has center **(10, −10)** and radius **1**.

2. a. In order to find center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to **complete the square**. Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$
$$x^2 + y^2 + 4x - 6y = \underline{3}$$
$$x^2 + 4x + \underline{4} + y^2 - 6y + \underline{9} = \underline{3} + \underline{4} + \underline{9}$$
$$(x + \underline{2})^2 + (y - \underline{3})^2 = \underline{16}$$

b. This circle has radius 4 and center at **(−2, 3)**.

Remember What You Learned

3. How can the distance formula help you to remember the equation of a circle?
Sample answer: Write the distance formula. Replace (x_1, y_1) with (h, k) and (x_2, y_2) with (x, y). Replace d with r. Square both sides. Now you have the equation of a circle.

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10-3 Study Guide and Intervention

Circles

Equations of Circles The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle if the endpoints of a diameter are at (−4, 5) and (6, −3).**

Use the midpoint formula to find the center of the circle.

$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint formula

$= \left(\frac{-4 + 6}{2}, \frac{5 + (-3)}{2}\right)$ (x_1, y_1) (−4, 5), (x_2, y_2) (6, −3)

$= \left(\frac{2}{2}, \frac{2}{2}\right)$ or (1, 1) Simplify.

Use the coordinates of the center and one endpoint of the diameter to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$r = \sqrt{(-4 - 1)^2 + (5 - 1)^2}$ (x_1, y_1) (1, 1), (x_2, y_2) (−4, 5)

$= \sqrt{(-5)^2 + 4^2} = \sqrt{41}$ Simplify.

The radius of the circle is $\sqrt{41}$, so $r^2 = 41$.

An equation of the circle is $(x - 1)^2 + (y - 1)^2 = 41$.

Exercises

Write an equation for the circle that satisfies each set of conditions.

1. center (8, −3), radius 6 **$(x - 8)^2 + (y + 3)^2 = 36$**

2. center (5, −6), radius 4 **$(x - 5)^2 + (y + 6)^2 = 16$**

3. center (−5, 2), passes through (−9, 6) **$(x + 5)^2 + (y - 2)^2 = 32$**

4. endpoints of a diameter at (6, 6) and (10, 12) **$(x - 8)^2 + (y - 9)^2 = 13$**

5. center (3, 6), tangent to the x-axis **$(x - 3)^2 + (y - 6)^2 = 36$**

6. center (−4, −7), tangent to $x = 2$ **$(x + 4)^2 + (y + 7)^2 = 36$**

7. center at (−2, 8), tangent to $y = -4$ **$(x + 2)^2 + (y - 8)^2 = 144$**

8. center (7, 7), passes through (12, 9) **$(x - 7)^2 + (y - 7)^2 = 29$**

9. endpoints of a diameter are (−4, −2) and (8, 4) **$(x - 2)^2 + (y - 1)^2 = 45$**

10. endpoints of a diameter are (−4, 3) and (6, −8) **$(x - 1)^2 + (y + 2.5)^2 = 55.25$**

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10-3 Study Guide and Intervention *(continued)*

Circles

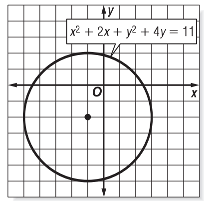
Graph Circles To graph a circle, write the given equation in the standard form of the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$.

Plot the center (h, k) of the circle. Then use r to calculate and plot the four points $(h + r, k)$, $(h - r, k)$, $(h, k + r)$, and $(h, k - r)$, which are all points on the circle. Sketch the circle that goes through those four points.

Example **Find the center and radius of the circle whose equation is $x^2 + 2x + y^2 + 4y = 11$. Then graph the circle.**

$$x^2 + 2x + y^2 + 4y = 11$$
$$x^2 + 2x + \blacksquare + y^2 + 4y + \blacksquare = 11 + \blacksquare$$
$$x^2 + 2x + 1 + y^2 + 4y + 4 = 11 + 1 + 4$$
$$(x + 1)^2 + (y + 2)^2 = 16$$

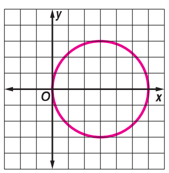
Therefore, the circle has its center at $(-1, -2)$ and a radius of $\sqrt{16} = 4$. Four points on the circle are $(3, -2)$, $(-5, -2)$, $(-1, 2)$, and $(-1, -6)$.



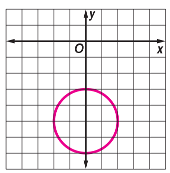
Exercises

Find the center and radius of the circle with the given equation. Then graph the circle.

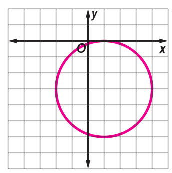
1. $(x - 3)^2 + y^2 = 9$
(3, 0), r = 3



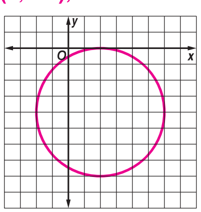
2. $x^2 + (y + 5)^2 = 4$
(0, −5), r = 2



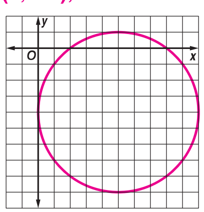
3. $(x - 1)^2 + (y + 3)^2 = 9$
(1, −3), r = 3



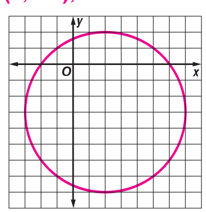
4. $(x - 2)^2 + (y + 4)^2 = 16$
(2, −4), r = 4



5. $x^2 + y^2 - 10x + 8y + 16 = 0$
(5, −4), r = 5



6. $x^2 + y^2 - 4x + 6y = 12$
(2, −3), r = 5



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10-3 Skills Practice

Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (0, 5), radius 1 unit
$x^2 + (y - 5)^2 = 1$

2. center (5, 12), radius 8 units
$(x - 5)^2 + (y - 12)^2 = 64$

3. center (4, 0), radius 2 units
$(x - 4)^2 + y^2 = 4$

4. center (2, 2), radius 3 units
$(x - 2)^2 + (y - 2)^2 = 9$

5. center (4, −4), radius 4 units
$(x - 4)^2 + (y + 4)^2 = 16$

6. center (−6, 4), radius 5 units
$(x + 6)^2 + (y - 4)^2 = 25$

7. endpoints of a diameter at (−12, 0) and (12, 0) **$x^2 + y^2 = 144$**

8. endpoints of a diameter at (−4, 0) and (−4, −6) **$(x + 4)^2 + (y + 3)^2 = 9$**

9. center at (7, −3), passes through the origin **$(x - 7)^2 + (y + 3)^2 = 58$**

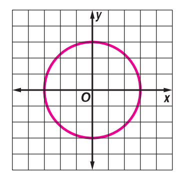
10. center at (−4, 4), passes through (−4, 1) **$(x + 4)^2 + (y - 4)^2 = 9$**

11. center at (−6, −5), tangent to y-axis **$(x + 6)^2 + (y + 5)^2 = 36$**

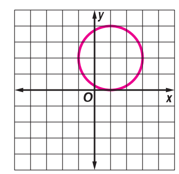
12. center at (5, 1), tangent to x-axis **$(x - 5)^2 + (y - 1)^2 = 1$**

Find the center and radius of the circle with the given equation. Then graph the circle.

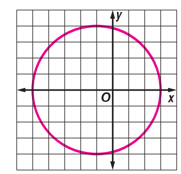
13. $x^2 + y^2 = 9$
(0, 0), 3 units



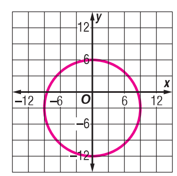
14. $(x - 1)^2 + (y - 2)^2 = 4$
(1, 2), 2 units



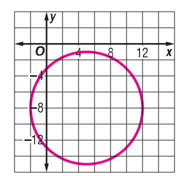
15. $(x + 1)^2 + y^2 = 16$
(−1, 0), 4 units



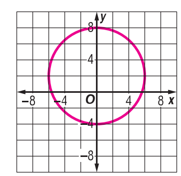
16. $x^2 + (y + 3)^2 = 81$
(0, −3), 9 units



17. $(x - 5)^2 + (y + 8)^2 = 49$
(5, −8), 7 units



18. $x^2 + y^2 - 4y - 32 = 0$
(0, 2), 6 units



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10-3 Practice
Circles

Write an equation for the circle that satisfies each set of conditions.

1. center (−4, 2), radius 8 units
$(x + 4)^2 + (y - 2)^2 = 64$

2. center (0, 0), radius 4 units
$x^2 + y^2 = 16$

3. center $\left(-\frac{1}{4}, -\sqrt{3}\right)$, radius $5\sqrt{2}$ units
$\left(x + \frac{1}{4}\right)^2 + (y + \sqrt{3})^2 = 50$

4. center (2.5, 4.2), radius 0.9 unit
$(x - 2.5)^2 + (y - 4.2)^2 = 0.81$

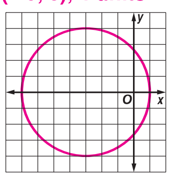
5. endpoints of a diameter at (−2, −9) and (0, −5) **$(x + 1)^2 + (y + 7)^2 = 5$**

6. center at (−9, −12), passes through (−4, −5) **$(x + 9)^2 + (y + 12)^2 = 74$**

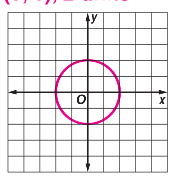
7. center at (−6, 5), tangent to x-axis **$(x + 6)^2 + (y - 5)^2 = 25$**

Find the center and radius of the circle with the given equation. Then graph the circle.

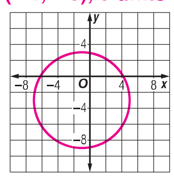
8. $(x + 3)^2 + y^2 = 16$
(−3, 0), 4 units



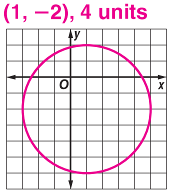
9. $3x^2 + 3y^2 = 12$
(0, 0), 2 units



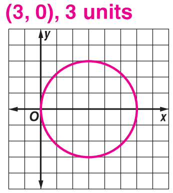
10. $x^2 + y^2 + 2x + 6y = 26$
(−1, −3), 6 units



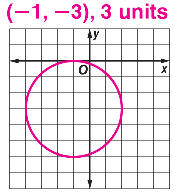
11. $(x - 1)^2 + y^2 + 4y = 12$
(1, −2), 4 units



12. $x^2 - 6x + y^2 = 0$
(3, 0), 3 units



13. $x^2 + y^2 + 2x + 6y = -1$
(−1, −3), 3 units



WEATHER For Exercises 14 and 15, use the following information.

On average, the circular eye of a hurricane is about 15 miles in diameter. Gale winds can affect an area up to 300 miles from the storm's center. In 2005, Hurricane Katrina devastated southern Louisiana. A satellite photo of Katrina's landfall showed the center of its eye on one coordinate system could be approximated by the point (80, 26).

14. Write an equation to represent a possible boundary of Katrina's eye.
$(x - 80)^2 + (y - 26)^2 = 56.25$

15. Write an equation to represent a possible boundary of the area affected by gale winds.
$(x - 80)^2 + (y - 26)^2 = 90,000$

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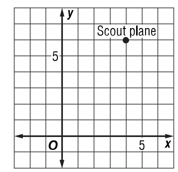
10-3 Word Problem Practice
Circles

1. RADAR A scout plane is equipped with radar. The boundary of the radar's range is given by the circle $(x - 4)^2 + (y - 6)^2 = 4900$. Each unit corresponds to one mile. What is the maximum distance that an object can be from the plane and still be detected by its radar?



70 mi

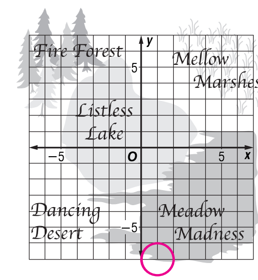
2. STORAGE An engineer uses a coordinate plane to show the layout of a side view of a storage building. The y-axis represents a wall and the x-axis represents the floor. A 10-meter diameter cylinder rests on its side flush against the wall. On the side view, the cylinder is represented by a circle in the first quadrant that is tangent to both axes. Each unit represents 1 meter. What is the equation of this circle?
$(x - 5)^2 + (y - 5)^2 = 25$

3. FERRIS WHEEL The Texas Star, the largest Ferris wheel in North America, is located in Dallas, Texas. It weighs 678,554 pounds and can hold 264 riders in its 44 gondolas. The Texas Star has a diameter of 212 feet. Use the rectangular coordinate system with the origin on the ground directly below the center of the wheel and write the equation of the circle that models the Texas Star.
$x^2 + (y - 106)^2 = 11,236$

4. POOLS The pool on an architectural floor plan is given by the equation $x^2 + 6x + y^2 + 8y = 0$. What point on the edge of the pool is farthest from the origin?
(−6, −8)

TREASURE For Exercises 5 and 6, use the following information.

A mathematically inclined pirate decided to hide the location of a treasure by marking it as the center of a circle given by an equation in non-standard form.



The secret circle can be represented by:

$$x^2 + y^2 - 2x + 14y + 49 = 0.$$

5. Rewrite the equation of the circle in standard form.
$(x - 1)^2 + (y + 7)^2 = 1$

6. Draw the circle on the map. Where is the treasure?
See circle on map at (1, −7); the southwest corner of Meadow Madness.

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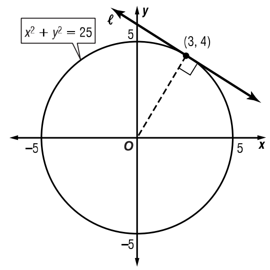
10-3 Enrichment

Tangents to Circles

A line that intersects a circle in exactly one point is a **tangent** to the circle. In the diagram, line ℓ is tangent to the circle with equation $x^2 + y^2 = 25$ at the point whose coordinates are (3, 4).

A line is tangent to a circle at a point P on the circle if and only if the line is perpendicular to the radius from the center of the circle to point P. This fact enables you to find an equation of the tangent to a circle at a point P if you know an equation for the circle and the coordinates of P.



Exercises

Use the diagram above to solve each problem.

1. What is the slope of the radius to the point with coordinates (3, 4)? What is the slope of the tangent to that point?

   $\frac{4}{3}$, $-\frac{3}{4}$

2. Find an equation of the line ℓ that is tangent to the circle at (3, 4).

   $y = -\frac{3}{4}x + \frac{25}{4}$

3. If k is a real number between -5 and 5, how many points on the circle have x-coordinate k? State the coordinates of these points in terms of k.

   two, $\left(k, \pm\sqrt{25 - k^2}\right)$

4. Describe how you can find equations for the tangents to the points you named for Exercise 3.

   Use the coordinates of (0, 0) and of one of the given points. Find the slope of the radius to that point. Use the slope of the radius to find what the slope of the tangent must be. Use the slope of the tangent and the coordinates of the point on the circle to find an equation for the tangent.

5. Find an equation for the tangent at (−3, 4).

   $y = \frac{3}{4}x + \frac{25}{x}$

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10-3 Graphing Calculator Activity

Matrices and Equations of Circles

A graphing calculator can be used to write the equation of a circle in the form $x^2 + y^2 + Dx + Ey + F = 0$ given any three points on the circle.

Example **Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.**

a. $A(5, 3)$, $B(-2, 2)$, and $C(-1, -5)$

Substitute each ordered pair for (x, y) in $x^2 + y^2 + Dx + Ey + F = 0$ to form the a system of equations.

$5D + 3E + F = -34$ $\quad -2D + 2E + F = -8$ $\quad -D - 5E + F = -26$

Solve the system using a matrix equation to find D, E, and F. Replace the coefficients in the expanded form. Then, complete the square to write the equation in standard form to identify the center and radius.

Keystrokes: 2nd [MATRX] ▶ ▶ ENTER 3 ENTER 3 ENTER 5 ENTER 3 ENTER 1 ENTER (–) 2 ENTER 2 ENTER 1 ENTER (–) 1 ENTER (–) 5 ENTER 1 ENTER 2nd [MATRX] ▶ ▶ [EDIT] 2 ENTER 3 ENTER 1 ENTER (–) 34 ENTER (–) 8 ENTER (–) 26 ENTER 2nd [QUIT] 2nd [MATRX] ENTER x^{-1} × 2nd [MATRX] 2 ENTER.

```
[A]⁻¹*[B]
          [[-4 ]
           [2  ]
           [-20]]
```

Thus, $D = -4$, $E = 2$, and $F = -20$.
The expanded form is $x^2 + y^2 - 4x + 2y - 20 = 0$.
After completing the square, the standard form is $(x - 2)^2 + (y + 1)^2 = 25$.
The center is (2, −1), and the radius is 5.

b. $A(-2, 3)$, $B(6, -5)$, and $C(0, 7)$

Find a system of equations. Then enter the equations into an *augmented matrix*. Reduce the matrix to *row reduced echelon form* using the **rref(** command. The row reduced echelon form of an augmented matrix will display the solution to the system.

$-2D + 3E + F = -13$ $\quad 6D - 5E + F = -61$ $\quad 7E + F = -49$

Keystrokes: Enter the system of equations as [A], a 3 × 4 augmented matrix. Then use the reduced row echelon form by pressing 2nd [MATRX] ▶ ALPHA [B] 2nd [MATRX] ENTER) ENTER.

```
[[-2  3  1  -13]
 [6  -5  1  -61]
 [0   7  1  -49]]
rref([A])
  [[1 0 0  -10]
   [0 1 0  -4 ]
   [0 0 1  -21]]
```

The solution is $D = -10$, $E = -4$, and $F = -21$. The expanded form is $x^2 + y^2 - 10x - 4y - 21 = 0$, standard form is $(x - 5)^2 + (y - 2)^2 = 50$. The center is (5, 2) and the radius is $5\sqrt{2}$.

Exercises

Write the equation of the circle that passes through the given points. Identify the center and radius of each circle.

1. (0, −1), (−3, −2), and (−6, −1)
   $x^2 + y^2 + 6x - 6y - 7 = 0$; $C(-3, 3)$, $R = 5$
2. (7, −1), (11, −5), and (3, −5)
   $x^2 + y^2 - 14x + 10y + 58 = 0$; $C(7, -5)$, $R = 4$
3. (−2, 7), (−9, 0), and (−10, −5)
   $x^2 + y^2 - 6x + 10y - 135 = 0$; $C(3, -5)$, $R = 13$

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10-4 Lesson Reading Guide

Ellipses

Get Ready for the Lesson

Read the introduction to Lesson 10-4 in your textbook.

Is the Earth always the same distance from the Sun? Explain your answer using the words *circle* and *ellipse*. **No; if the Earth's orbit were a circle, it would always be the same distance from the Sun because every point on a circle is the same distance from the center. However, the Earth's orbit is an ellipse, and the points on an ellipse are not all the same distance from the center.**

Read the Lesson

1. An ellipse is the set of all points in a plane such that the **sum** of the distances from two fixed points is **constant**. The two fixed points are called the **foci** of the ellipse.

2. Consider the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

a. For this equation, a = **3** and b = **2**.

b. Write an equation that relates the values of a, b, and c. $c^2 = a^2 - b^2$

c. Find the value of c for this ellipse. $\sqrt{5}$

3. Consider the ellipses with equations $\frac{y^2}{25} + \frac{x^2}{16} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Complete the following table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis	vertical	horizontal
Direction of Minor Axis	horizontal	vertical
Foci	(0, 3), (0, −3)	$(\sqrt{5}, 0), (-\sqrt{5}, 0)$
Length of Major Axis	10 units	6 units
Length of Minor Axis	8 units	4 units

Remember What You Learned

4. Some students have trouble remembering the two standard forms for the equation of an ellipse. How can you remember which term comes first and where to place a and b in these equations? **The *x*-axis is horizontal. If the major axis is horizontal, the first term is $\frac{x^2}{a^2}$. The *y*-axis is vertical. If the major axis is vertical, the first term is $\frac{y^2}{a^2}$. *a* is always the larger of the numbers *a* and *b*.**

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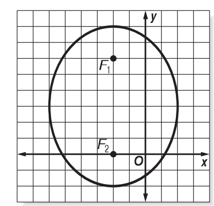
10-4 Study Guide and Intervention

Ellipses

Equations of Ellipses An **ellipse** is the set of all points in a plane such that the *sum* of the distances from two given points in the plane, called the foci, is constant. An ellipse has two axes of symmetry which contain the **major** and **minor axes**. In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 - b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Direction of Major Axis	Horizontal	Vertical
Foci	$(h + c, k), (h - c, k)$	$(h, k - c), (h, k + c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

Example **Write an equation for the ellipse shown.**



The length of the major axis is the distance between (−2, −2) and (−2, 8). This distance is 10 units.

$2a = 10$, so $a = 5$

The foci are located at (−2, 6) and (−2, 0), so $c = 3$.

$b^2 = a^2 - c^2$
$= 25 - 9$
$= 16$

The center of the ellipse is at (−2, 3), so $h = -2$, $k = 3$, $a^2 = 25$, and $b^2 = 16$. The major axis is vertical.

An equation of the ellipse is $\frac{(y-3)^2}{25} + \frac{(x+2)^2}{16} = 1$.

Exercises

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at (−7, 2) and (5, 2), endpoints of minor axis at (−1, 0) and (−1, 4)
$\frac{(x+1)^2}{36} + \frac{(y-2)^2}{4} = 1$

2. major axis 8 units long and parallel to the x-axis, minor axis 2 units long, center at (−2, −5)
$\frac{(x+2)^2}{16} + (y+5)^2 = 1$

3. endpoints of major axis at (−8, 4) and (4, 4), foci at (−3, 4) and (−1, 4)
$\frac{(x+2)^2}{36} + \frac{(y-4)^2}{35} = 1$

4. endpoints of major axis at (3, 2) and (3, −14), endpoints of minor axis at (−1, −6) and (7, −6)
$\frac{(y+6)^2}{64} + \frac{(x-3)^2}{16} = 1$

5. minor axis 6 units long and parallel to the x-axis, major axis 12 units long, center at (6, 1)
$\frac{(y-1)^2}{36} + \frac{(x-6)^2}{9} = 1$

Lesson 10-4

Answers

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10-4 Study Guide and Intervention *(continued)*

Ellipses

Graph Ellipses To graph an ellipse, if necessary, write the given equation in the standard form of an equation for an ellipse.

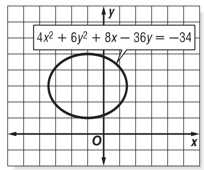
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (for ellipse with major axis horizontal) or

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ (for ellipse with major axis vertical)

Use the center (h, k) and the endpoints of the axes to plot four points of the ellipse. To make a more accurate graph, use a calculator to find some approximate values for x and y that satisfy the equation.

Example **Graph the ellipse $4x^2 + 6y^2 + 8x - 36y = -34$.**

$$4x^2 + 6y^2 + 8x - 36y = -34$$
$$4x^2 + 8x + 6y^2 - 36y = -34$$
$$4(x^2 + 2x + \blacksquare) + 6(y^2 - 6y + \blacksquare) = -34 + \blacksquare$$
$$4(x^2 + 2x + 1) + 6(y^2 - 6y + 9) = -34 + 58$$
$$4(x + 1)^2 + 6(y - 3)^2 = 24$$
$$\frac{(x+1)^2}{6} + \frac{(y-3)^2}{4} = 1$$



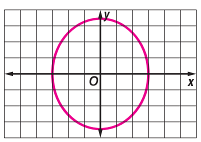
The center of the ellipse is $(-1, 3)$. Since $a^2 = 6$, $a = \sqrt{6}$. Since $b^2 = 4$, $b = 2$.

The length of the major axis is $2\sqrt{6}$, and the length of the minor axis is 4. Since the x-term has the greater denominator, the major axis is horizontal. Plot the endpoints of the axes. Then graph the ellipse.

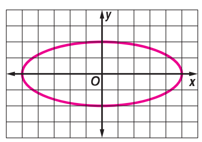
Exercises

Find the coordinates of the center and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

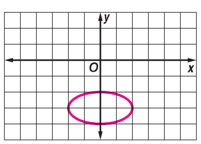
1. $\frac{y^2}{12} + \frac{x^2}{9} = 1$ **(0, 0), $4\sqrt{3}$, 6**



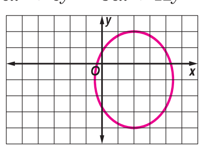
2. $\frac{x^2}{25} + \frac{y^2}{4} = 1$ **(0, 0), 10, 4**



3. $x^2 + 4y^2 + 24y = -32$ **(0, −3), 4, 2**



4. $9x^2 + 6y^2 - 36x + 12y = 12$ **(2, −1), 6, $2\sqrt{6}$**



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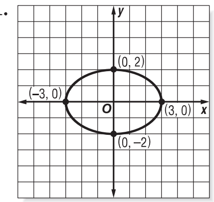
10-4 Skills Practice

Ellipses

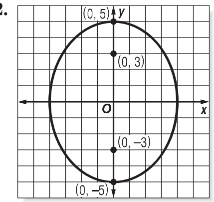
Write an equation for each ellipse.

1.



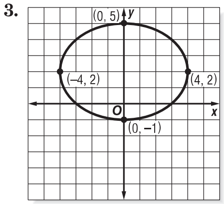
$\frac{x^2}{9} + \frac{y^2}{4} = 1$

2.



$\frac{y^2}{25} + \frac{x^2}{16} = 1$

3.



$\frac{x^2}{16} + \frac{(y-2)^2}{9} = 1$

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at (0, 6) and (0, −6), endpoints of minor axis at (−3, 0) and (3, 0)
$\frac{y^2}{36} + \frac{x^2}{9} = 1$

5. endpoints of major axis at (2, 6) and (8, 6), endpoints of minor axis at (5, 4) and (5, 8)
$\frac{(x-5)^2}{9} + \frac{(y-6)^2}{4} = 1$

6. endpoints of major axis at (7, 3) and (7, 9), endpoints of minor axis at (5, 6) and (9, 6)
$\frac{(y-6)^2}{9} + \frac{(x-7)^2}{4} = 1$

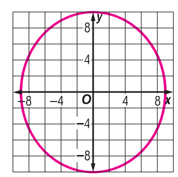
7. major axis 12 units long and parallel to x-axis, minor axis 4 units long, center at (0, 0)
$\frac{x^2}{36} + \frac{y^2}{4} = 1$

8. endpoints of major axis at (−6, 0) and (6, 0), foci at $(-\sqrt{32}, 0)$ and $(\sqrt{32}, 0)$
$\frac{x^2}{36} + \frac{y^2}{4} = 1$

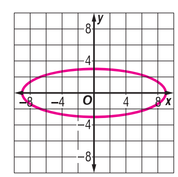
9. endpoints of major axis at (0, 12) and (0, −12), foci at $(0, \sqrt{23})$ and $(0, -\sqrt{23})$
$\frac{y^2}{144} + \frac{x^2}{121} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

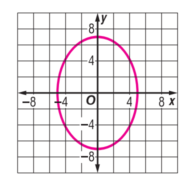
10. $\frac{y^2}{100} + \frac{x^2}{81} = 1$
(0, 0); $(0, \pm\sqrt{19})$; 20; 18



11. $\frac{x^2}{81} + \frac{y^2}{9} = 1$
(0, 0); $(\pm 6\sqrt{2}, 0)$; 18; 6



12. $\frac{y^2}{49} + \frac{x^2}{25} = 1$
(0, 0), $(0, \pm 2\sqrt{6})$; 14; 10



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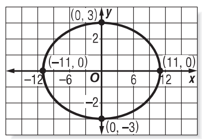
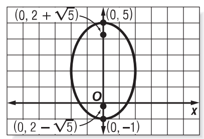
10-4 Practice

Ellipses

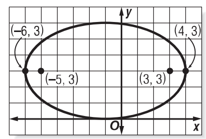
Write an equation for each ellipse.

1. 

$\frac{x^2}{121} + \frac{y^2}{9} = 1$

2. 

$\frac{(y-2)^2}{9} + \frac{x^2}{4} = 1$

3. 

$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(-9, 0)$ and $(9, 0)$, endpoints of minor axis at $(0, 3)$ and $(0, -3)$
$\frac{x^2}{81} + \frac{y^2}{9} = 1$

5. endpoints of major axis at $(4, 2)$ and $(4, -8)$, endpoints of minor axis at $(1, -3)$ and $(7, -3)$
$\frac{(y+3)^2}{25} + \frac{(x-4)^2}{9} = 1$

6. major axis 20 units long and parallel to x-axis, minor axis 10 units long, center at $(2, 1)$
$\frac{(x-2)^2}{100} + \frac{(y-1)^2}{25} = 1$

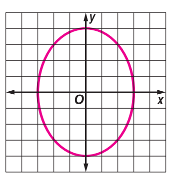
7. major axis 10 units long, minor axis 6 units long and parallel to x-axis, center at $(2, -4)$
$\frac{(y+4)^2}{25} + \frac{(x-2)^2}{9} = 1$

8. major axis 16 units long, center at $(0, 0)$, foci at $(0, 2\sqrt{15})$ and $(0, -2\sqrt{15})$
$\frac{y^2}{64} + \frac{x^2}{4} = 1$

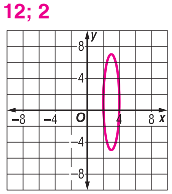
9. endpoints of minor axis at $(0, 2)$ and $(0, -2)$, foci at $(-4, 0)$ and $(4, 0)$
$\frac{x^2}{20} + \frac{y^2}{4} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

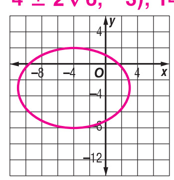
10. $\frac{y^2}{16} + \frac{x^2}{9} = 1$
$(0, 0)$; $(0, \pm\sqrt{7})$; 8; 6



11. $\frac{(y-1)^2}{36} + \frac{(x-3)^2}{1} = 1$
$(3, 1)$; $(3, 1 \pm \sqrt{35})$; 12; 2



12. $\frac{(x+4)^2}{49} + \frac{(y+3)^2}{25} = 1$
$(-4, -3)$; $(-4 \pm 2\sqrt{6}, -3)$; 14; 10



13. **SPORTS** An ice skater traces two congruent ellipses to form a figure eight. Assume that the center of the first loop is at the origin, with the second loop to its right. Write an equation to model the first loop if its major axis (along the x-axis) is 12 feet long and its minor axis is 6 feet long. Write another equation to model the second loop.
$\frac{x^2}{36} + \frac{y^2}{9} = 1$; $\frac{(x-12)^2}{36} + \frac{y^2}{9} = 1$

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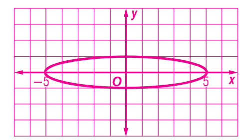
10-4 Word Problem Practice

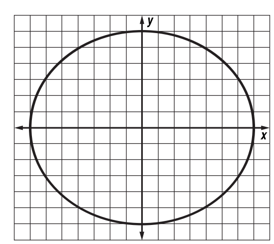
Ellipses

1. **PERSPECTIVE** A graphic designer uses an ellipse to draw a circle from the horizontal perspective. The equation used is $\frac{x^2}{25} + y^2 = 1$. Graph this ellipse.



2. **ECHOES** The walls of an elliptical room are given by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Two people want to stand at the foci of the ellipse so that they can whisper to each other without anybody else hearing. What are the coordinates of the foci?
$(3, 0)$ and $(-3, 0)$

3. **FLASHLIGHTS** Daniella ended up doing her math homework late at night. To avoid disturbing others, she worked in bed with a pen light. One problem asked her to draw an ellipse. She noticed that her pen light created an elliptical patch of light on her paper, so she simply traced the outline of the patch of light. The outline of the ellipse is shown below. What is the equation of this ellipse in standard form?



$\frac{x^2}{49} + \frac{y^2}{36} = 1$

4. **ASTRONOMY** The orbit of an asteroid is given by the equation $\frac{x^2}{400} + \frac{y^2}{441} = 1$, where each unit represents one astronomical unit (i.e. the distance from Sun to Earth). What are the lengths of the major and minor axes of the orbit?
Major axis: 42 astronomical units, Minor axis: 40 astronomical units

MODELING For Exercises 5 and 6, use the following information.

James wants to try to make an ellipse using a piece of string 26 inches long. He tacks the two ends down 10 inches apart. He then takes a pen and pulls the string taut. He keeps the string taut and pulls the pen around the tacks. By doing this, he creates an ellipse.

5. Determine the lengths of the major and minor axes of the ellipse that James drew.
Major axis has length 26, minor axis has length 24.

6. If a coordinate grid is overlaid on the ellipse so that the tacks are located at $(5, 0)$ and $(-5, 0)$, what is the equation of the ellipse in standard form?
$\frac{x^2}{169} + \frac{y^2}{144} = 1$

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10-4 Enrichment

Eccentricity

In an ellipse, the ratio $\frac{c}{a}$ is called the **eccentricity** and is denoted by the letter e. Eccentricity measures the elongation of an ellipse. The closer e is to 0, the more an ellipse looks like a circle. The closer e is to 1, the more elongated it is. Recall that the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where a is the length of the major axis, and that $c = \sqrt{a^2 - b^2}$.

Find the eccentricity of each ellipse rounded to the nearest hundredth.

1. $\frac{x^2}{9} + \frac{y^2}{36} = 1$ **0.87**

2. $\frac{x^2}{81} + \frac{y^2}{9} = 1$ **0.94**

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **0.75**

4. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ **0.66**

5. $\frac{x^2}{36} + \frac{y^2}{16} = 1$ **0.75**

6. $\frac{x^2}{4} + \frac{y^2}{36} = 1$ **0.94**

7. Is a circle an ellipse? Explain your reasoning.

Yes; it is an ellipse with eccentricity 0.

8. The center of the sun is one focus of Earth's orbit around the sun. The length of the major axis is 186,000,000 miles, and the foci are 3,200,000 miles apart. Find the eccentricity of Earth's orbit.

approximately 0.017

9. An artificial satellite orbiting the earth travels at an altitude that varies between 132 miles and 583 miles above the surface of the earth. If the center of the earth is one focus of its elliptical orbit and the radius of the earth is 3950 miles, what is the eccentricity of the orbit?

approximately 0.052

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10-5 Lesson Reading Guide

Hyperbolas

Get Ready for the Lesson

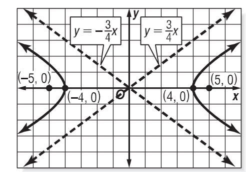
Read the introduction to Lesson 10-5 in your textbook.

Look at the sketch of a hyperbola in the introduction to this lesson. List three ways in which hyperbolas are different from parabolas.

Sample answer: A hyperbola has two branches, while a parabola is one continuous curve. A hyperbola has two foci, while a parabola has one focus. A hyperbola has two vertices, while a parabola has one vertex.

Read the Lesson

1. The graph at the right shows the hyperbola whose equation in standard form is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



The point (0, 0) is the __**center**__ of the hyperbola.

The points (4, 0) and (−4, 0) are the __**vertices**__ of the hyperbola.

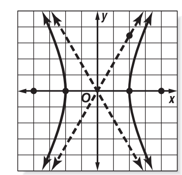
The points (5, 0) and (−5, 0) are the __**foci**__ of the hyperbola.

The segment connecting (4, 0) and (−4, 0) is called the __**transverse**__ axis.

The segment connecting (0, 3) and (0, −3) is called the __**conjugate**__ axis.

The lines $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$ are called the __**asymptotes**__.

2. Study the hyperbola graphed at the right.



The center is __**(0, 0)**__.

The value of a is __**2**__.

The value of c is __**4**__.

To find b^2, solve the equation __c^2__ = __a^2__ + __b^2__.

The equation in standard form for this hyperbola is __$\frac{x^2}{4} - \frac{y^2}{12} = 1$__.

Remember What You Learned

3. What is an easy way to remember the equation relating the values of a, b, and c for a hyperbola? **This equation looks just like the Pythagorean Theorem, although the variables represent different lengths in a hyperbola than in a right triangle.**

Lesson 10-5

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10-5 Study Guide and Intervention

Hyperbolas

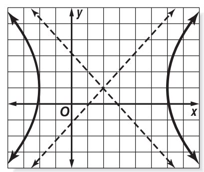
Equations of Hyperbolas A **hyperbola** is the set of all points in a plane such that the absolute value of the *difference* of the distances from any point on the hyperbola to any two given points in the plane, called the **foci**, is constant.

In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 + b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

Example **Write an equation for the hyperbola with vertices (−2, 1) and (6, 1) and foci (−4, 1) and (8, 1).**

Use a sketch to orient the hyperbola correctly. The center of the hyperbola is the midpoint of the segment joining the two vertices. The center is $(\frac{-2+6}{2}, 1)$, or $(2, 1)$. The value of a is the distance from the center to a vertex, so $a = 4$. The value of c is the distance from the center to a focus, so $c = 6$.



$c^2 = a^2 + b^2$
$6^2 = 4^2 + b^2$
$b^2 = 36 - 16 = 20$

Use h, k, a^2, and b^2 to write an equation of the hyperbola.

$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{20} = 1$

Exercises

Write an equation for the hyperbola that satisfies each set of conditions.

1. vertices $(-7, 0)$ and $(7, 0)$, conjugate axis of length 10 $\mathbf{\frac{x^2}{49} - \frac{y^2}{25} = 1}$

2. vertices $(-2, -3)$ and $(4, -3)$, foci $(-5, -3)$ and $(7, -3)$ $\mathbf{\frac{(x-1)^2}{9} - \frac{(y+3)^2}{27} = 1}$

3. vertices $(4, 3)$ and $(4, -5)$, conjugate axis of length 4 $\mathbf{\frac{(y+1)^2}{16} - \frac{(x-4)^2}{4} = 1}$

4. vertices $(-8, 0)$ and $(8, 0)$, equation of asymptotes $y = \pm\frac{1}{6}x$ $\mathbf{\frac{x^2}{64} - \frac{9y^2}{16} = 1}$

5. vertices $(-4, 6)$ and $(-4, -2)$, foci $(-4, 10)$ and $(-4, -6)$ $\mathbf{\frac{(y-2)^2}{16} - \frac{(x+4)^2}{48} = 1}$

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10-5 Study Guide and Intervention *(continued)*

Hyperbolas

Graph Hyperbolas To graph a hyperbola, write the given equation in the standard form of an equation for a hyperbola

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ if the branches of the hyperbola open left and right, or

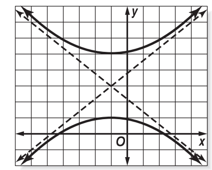
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ if the branches of the hyperbola open up and down

Graph the point (h, k), which is the center of the hyperbola. Draw a rectangle with dimensions $2a$ and $2b$ and center (h, k). If the hyperbola opens left and right, the vertices are $(h - a, k)$ and $(h + a, k)$. If the hyperbola opens up and down, the vertices are $(h, k - a)$ and $(h, k + a)$.

Example **Draw the graph of $6y^2 - 4x^2 - 36y - 8x = -26$.**

Complete the squares to get the equation in standard form.

$6y^2 - 4x^2 - 36y - 8x = -26$
$6(y^2 - 6y + ■) - 4(x^2 + 2x + ■) = -26 + ■$
$6(y^2 - 6y + 9) - 4(x^2 + 2x + 1) = -26 + 50$
$6(y - 3)^2 - 4(x + 1)^2 = 24$
$\frac{(y-3)^2}{4} - \frac{(x+1)^2}{6} = 1$



The center of the hyperbola is $(-1, 3)$.

According to the equation, $a^2 = 4$ and $b^2 = 6$, so $a = 2$ and $b = \sqrt{6}$.

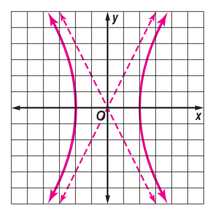
The transverse axis is vertical, so the vertices are $(-1, 5)$ and $(-1, 1)$. Draw a rectangle with vertical dimension 4 and horizontal dimension $2\sqrt{6} \approx 4.9$. The diagonals of this rectangle are the asymptotes. The branches of the hyperbola open up and down. Use the vertices and the asymptotes to sketch the hyperbola.

Exercises

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

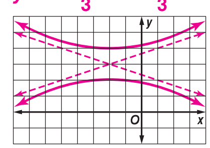
1. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

(2, 0), (−2, 0); $(2\sqrt{5}, 0), (-2\sqrt{5}, 0)$; $y = \pm 2x$



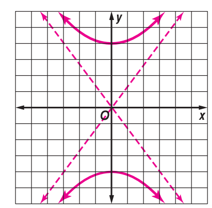
2. $(y - 3)^2 - \frac{(x+2)^2}{9} = 1$

(−2, 4), (−2, 2); $(-2, 3 + \sqrt{10})$, $(-2, 3 - \sqrt{10})$; $y = \frac{1}{3}x + 3\frac{2}{3}$, $y = -\frac{1}{3}x + 2\frac{1}{3}$



3. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

(0, 4), (0, −4); (0, 5), (0, −5); $y = \pm\frac{4}{3}x$



Lesson 10-5

Answers

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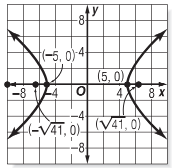
10-5 Skills Practice

Hyperbolas

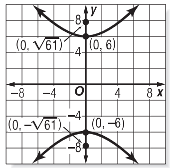
Write an equation for each hyperbola.

1.



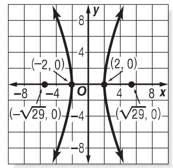
$\frac{x^2}{25} - \frac{y^2}{16} = 1$

2.



$\frac{y^2}{36} - \frac{x^2}{25} = 1$

3.



$\frac{x^2}{4} - \frac{y^2}{25} = 1$

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices (−4, 0) and (4, 0), conjugate axis of length 8 $\frac{x^2}{16} - \frac{y^2}{16} = 1$

5. vertices (0, 6) and (0, −6), conjugate axis of length 14 $\frac{y^2}{36} - \frac{x^2}{49} = 1$

6. vertices (0, 3) and (0, −3), conjugate axis of length 10 $\frac{y^2}{9} - \frac{x^2}{25} = 1$

7. vertices (−2, 0) and (2, 0), conjugate axis of length 4 $\frac{x^2}{4} - \frac{y^2}{4} = 1$

8. vertices (−3, 0) and (3, 0), foci (±5, 0) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

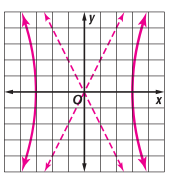
9. vertices (0, 2) and (0, −2), foci (0, ±3) $\frac{y^2}{4} - \frac{x^2}{5} = 1$

10. vertices (0, −2) and (6, −2), foci $(3 \pm \sqrt{13}, -2)$ $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

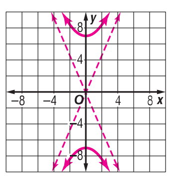
11. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

(±3, 0); $(\pm 3\sqrt{5}, 0)$;
$y = \pm 2x$



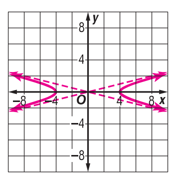
12. $\frac{y^2}{49} - \frac{x^2}{9} = 1$

(0, ±7); $(0, \pm\sqrt{58})$;
$y = \pm\frac{7}{3}x$



13. $\frac{x^2}{16} - \frac{y^2}{1} = 1$

(±4, 0); $(\pm\sqrt{17}, 0)$;
$y = \pm\frac{1}{4}x$



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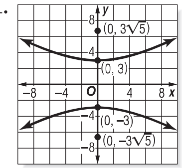
10-5 Practice

Hyperbolas

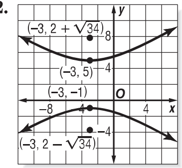
Write an equation for each hyperbola.

1.



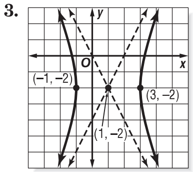
$\frac{y^2}{9} - \frac{x^2}{36} = 1$

2.



$\frac{(y-2)^2}{9} - \frac{(x+3)^2}{25} = 1$

3.



$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices (0, 7) and (0, −7), conjugate axis of length 18 units $\frac{y^2}{49} - \frac{x^2}{81} = 1$

5. vertices (1, −1) and (1, −9), conjugate axis of length 6 units $\frac{(y+5)^2}{16} - \frac{(x-1)^2}{9} = 1$

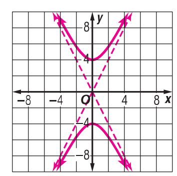
6. vertices (−5, 0) and (5, 0), foci $(\pm\sqrt{26}, 0)$ $\frac{x^2}{25} - \frac{y^2}{1} = 1$

7. vertices (1, 1) and (1, −3), foci $(1, -1 \pm \sqrt{5})$ $\frac{(y+1)^2}{4} - \frac{(x-1)^2}{1} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

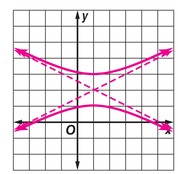
8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

(0, ±4); $(0, \pm 2\sqrt{5})$;
$y = \pm 2x$



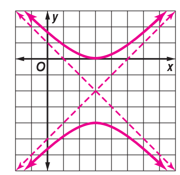
9. $\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$

(1, 3), (1, 1);
$(1, 2 \pm \sqrt{5})$;
$y - 2 = \pm\frac{1}{2}(x - 1)$



10. $\frac{(y+2)^2}{4} - \frac{(x-3)^2}{4} = 1$

(3, 0), (3, −4);
$(3, -2 \pm 2\sqrt{2})$;
$y + 2 = \pm(x - 3)$



11. **ASTRONOMY** Astronomers use special X-ray telescopes to observe the sources of celestial X rays. Some X-ray telescopes are fitted with a metal mirror in the shape of a hyperbola, which reflects the X rays to a focus. Suppose the vertices of such a mirror are located at (−3, 0) and (3, 0), and one focus is located at (5, 0). Write an equation that models the hyperbola formed by the mirror. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

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Lesson 10-5

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10-5 Word Problem Practice

Hyperbolas

1. **LIGHTHOUSES** The location of a lighthouse is represented by the origin of a coordinate plane. A boat in the distance appears to be on a collision course with the lighthouse. However, the boat veers off and turns away at the last moment, avoiding the rocky shallows. The path followed by the boat is modeled by a branch of the hyperbola with equation $\frac{x^2}{900} - \frac{y^2}{400} = 1$. If the unit length corresponds to a yard, how close did the boat come to the lighthouse?
30 yd

2. **FIND THE ERROR** Curtis was trying to write the equation for a hyperbola with a vertical transverse axis of length 10 and conjugate axis of length 6. The equation he got was $\frac{y^2}{9} - \frac{x^2}{25} = 1$. Did he make a mistake? If so, what did he do wrong?
Yes. Curtis exchanged the transverse and conjugate axes.

3. **MIRROR** At a carnival, designers are planning a funhouse. They plan to put a large hyperbolic mirror inside this funhouse. They design the mirror's hyperbolic cross section on graph paper using a hyperbola with a horizontal transverse axis. The asymptotes are to be $y = 9x$ and $y = -9x$ so the mirror is somewhat shallow. They also want the vertices to be 1 unit from the origin. What equation should they use for the hyperbola?
$\frac{x^2}{1} - \frac{y^2}{1} = 1$

4. **ASTRONOMY** Astronomers discover a new comet. They study its path and discover that it can be modeled by a branch of a hyperbola with equation $4x^2 - 40x - 25y^2 = 0$. Rewrite this equation in standard form and find the center of the hyperbola.
$\frac{(x-5)^2}{25} - \frac{y^2}{4} = 1$, **center at (5, 0)**

LIGHTNING For Exercises 5–7, use the following information.

Brittany and Kirk were talking on the phone when Brittany heard the thunder from a lightning bolt outside. Eight seconds later, she could hear the same thunder over the phone. Brittany and Kirk live 2 miles apart and sounds travels about 1 mile every 5 seconds.

5. On a coordinate plane, assume that Brittany is located at $(-1, 0)$ and Kirk is located at $(1, 0)$. Write an equation using the Distance Formula that describes the possible locations of the lightning strike.
$\sqrt{(x-1)^2 + y^2} - \sqrt{(x+1)^2 + y^2} = \frac{8}{5}$

6. Rewrite the equation you wrote for Exercise 5 so it is in the standard form for a hyperbola.
$\frac{x^2}{\left(\frac{4}{5}\right)^2} - \frac{y^2}{\left(\frac{3}{5}\right)^2} = 1$

7. Which branch of the hyperbola corresponds to the places where the lightning bolt might have struck?
the left branch, the branch with negative x coordinates

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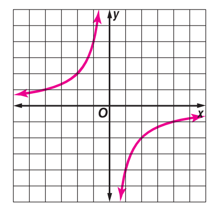
10-5 Enrichment

Rectangular Hyperbolas

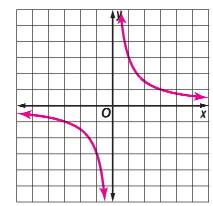
A **rectangular hyperbola** is a hyperbola with perpendicular asymptotes. For example, the graph of $x^2 - y^2 = 1$ is a rectangular hyperbola. A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. The graphs of equations of the form $xy = c$, where c is a constant, are rectangular hyperbolas.

Make a table of values and plot points to graph each rectangular hyperbola below. Be sure to consider negative values for the variables. **See students' tables.**

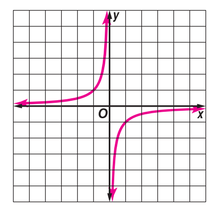
1. $xy = -4$



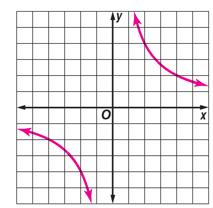
2. $xy = 3$



3. $xy = -1$



4. $xy = 8$



5. Make a conjecture about the asymptotes of rectangular hyperbolas.
The coordinate axes are the asymptotes.

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10-6 Lesson Reading Guide

Conic Sections

Get Ready for the Lesson

Read the introduction to Lesson 10-6 in your textbook.

The figures in the introduction show how a plane can slice a double cone to form the conic sections. Name the conic section that is formed if the plane slices the double cone in each of the following ways:

- The plane is parallel to the base of the double cone and slices through one of the cones that form the double cone. **circle**
- The plane is perpendicular to the base of the double cone and slices through both of the cones that form the double cone. **hyperbola**

Read the Lesson

1. Name the conic section that is the graph of each of the following equations. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.

   a. $\frac{(x-3)^2}{36} + \frac{(y+5)^2}{15} = 1$ **ellipse; (3, –5)**

   b. $x = -2(y+1)^2 + 7$ **parabola; (7, –1)**

   c. $(x-5)^2 - (y+5)^2 = 1$ **hyperbola; (5, –5)**

   d. $(x+6)^2 + (y-2)^2 = 1$ **circle; (–6, 2)**

2. Each of the following is the equation of a conic section. For each equation, identify the values of A and C. Then, without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

   a. $2x^2 + y^2 - 6x + 8y + 12 = 0$ $A =$ **2**; $C =$ **1**; type of graph: **ellipse**

   b. $2x^2 + 3x - 2y - 5 = 0$ $A =$ **2**; $C =$ **0**; type of graph: **parabola**

   c. $5x^2 + 10x + 5y^2 - 20y + 1 = 0$ $A =$ **5**; $C =$ **5**; type of graph: **circle**

   d. $x^2 - y^2 + 4x + 2y - 5 = 0$ $A =$ **1**; $C =$ **–1**; type of graph: **hyperbola**

Remember What You Learned

3. What is an easy way to recognize that an equation represents a parabola rather than one of the other conic sections?

   If the equation has an x^2 term and y term but no y^2 term, then the graph is a parabola. Likewise, if the equation has a y^2 term and x term but no x^2 term, then the graph is a parabola.

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10-6 Study Guide and Intervention

Conic Sections

Standard Form Any conic section in the coordinate plane can be described by an equation of the form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C are not all zero.

One way to tell what kind of conic section an equation represents is to rearrange terms and complete the square, if necessary, to get one of the standard forms from an earlier lesson. This method is especially useful if you are going to graph the equation.

Example **Write the equation $3x^2 - 4y^2 - 30x - 8y + 59 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

$3x^2 - 4y^2 - 30x - 8y + 59 = 0$	Original equation
$3x^2 - 30x - 4y^2 - 8y = -59$	Isolate terms.
$3(x^2 - 10x + ■) - 4(y^2 + 2y + ■) = -59 + ■ + ■$	Factor out common multiples.
$3(x^2 - 10x + 25) - 4(y^2 + 2y + 1) = -59 + 3(25) + (-4)(1)$	Complete the squares.
$3(x-5)^2 - 4(y+1)^2 = 12$	Simplify.
$\frac{(x-5)^2}{4} - \frac{(y+1)^2}{3} = 1$	Divide each side by 12.

The graph of the equation is a hyperbola with its center at (5, −1). The length of the transverse axis is 4 units and the length of the conjugate axis is $2\sqrt{3}$ units.

Exercises

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 + y^2 - 6x + 4y + 3 = 0$
   $(x-3)^2 + (y+2)^2 = 10$; circle

2. $x^2 + 2y^2 + 6x - 20y + 53 = 0$
   $\frac{(x+3)^2}{6} + \frac{(y-5)^2}{3} = 1$; ellipse

3. $6x^2 - 60x - y + 161 = 0$
   $y = 6(x-5)^2 + 11$; parabola

4. $x^2 + y^2 - 4x - 14y + 29 = 0$
   $(x-2)^2 + (y-7)^2 = 24$; circle

5. $6x^2 - 5y^2 + 24x + 20y - 56 = 0$
   $\frac{(x+2)^2}{10} - \frac{(y-2)^2}{12} = 1$; hyperbola

6. $3y^2 + x - 24y + 46 = 0$
   $x = -3(y-4)^2 + 2$; parabola

7. $x^2 - 4y^2 - 16x + 24y - 36 = 0$
   $\frac{(x-8)^2}{64} - \frac{(y-3)^2}{16} = 1$; hyperbola

8. $x^2 + 2y^2 + 8x + 4y + 2 = 0$
   $\frac{(x+4)^2}{16} + \frac{(y+1)^2}{8} = 1$; ellipse

9. $4x^2 + 48x + y + 158 = 0$
   $y = -4(x+6)^2 - 14$; parabola

10. $3x^2 + y^2 - 48x - 4y + 184 = 0$
    $\frac{(x-8)^2}{4} + \frac{(y-2)^2}{12} = 1$; ellipse

11. $-3x^2 + 2y^2 - 18x + 20y + 5 = 0$
    $\frac{(y+5)^2}{9} - \frac{(x+3)^2}{6} = 1$; hyperbola

12. $x^2 + y^2 + 8x + 2y + 8 = 0$
    $(x+4)^2 + (y+1)^2 = 9$; circle

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10-6 Study Guide and Intervention *(continued)*

Conic Sections

Identify Conic Sections If you are given an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \text{ with } B = 0,$$

you can determine the type of conic section just by considering the values of A and C. Refer to the following chart.

Relationship of A and C	Type of Conic Section
$A = 0$ or $C = 0$, but not both.	parabola
$A = C$	circle
A and C have the same sign, but $A \neq C$.	ellipse
A and C have opposite signs.	hyperbola

Example **Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

a. $3x^2 - 3y^2 + 5x + 12 = 0$

$A = 3$ and $C = -3$ have opposite signs, so the graph of the equation is a hyperbola.

b. $y^2 = 7y - 2x + 13$

$A = 0$, so the graph of the equation is a parabola.

Exercises

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 = 17x - 5y + 8$ **parabola**
2. $2x^2 + 2y^2 - 3x + 4y = 5$ **circle**
3. $4x^2 - 8x = 4y^2 - 6y + 10$ **hyperbola**
4. $8(x - x^2) = 4(2y^2 - y) - 100$ **circle**
5. $6y^2 - 18 = 24 - 4x^2$ **ellipse**
6. $y = 27x - y^2$ **parabola**
7. $x^2 = 4(y - y^2) + 2x - 1$ **ellipse**
8. $10x - x^2 - 2y^2 = 5y$ **ellipse**
9. $x = y^2 - 5y + x^2 - 5$ **circle**
10. $11x^2 - 7y^2 = 77$ **hyperbola**
11. $3x^2 + 4y^2 = 50 + y^2$ **circle**
12. $y^2 = 8x - 11$ **parabola**
13. $9y^2 - 99y = 3(3x - 3x^2)$ **circle**
14. $6x^2 - 4 = 5y^2 - 3$ **hyperbola**
15. $111 = 11x^2 + 10y^2$ **ellipse**
16. $120x^2 - 119y^2 + 118x - 117y = 0$ **hyperbola**
17. $3x^2 = 4y^2 + 12$ **hyperbola**
18. $150 - x^2 = 120 - y$ **parabola**

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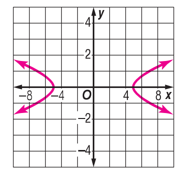
10-6 Skills Practice

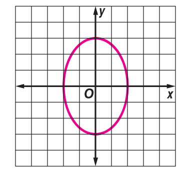
Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

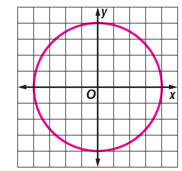
1. $x^2 - 25y^2 = 25$ **hyperbola** $\frac{x^2}{25} - \frac{y^2}{1} = 1$



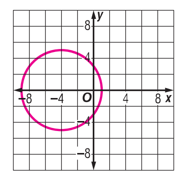
2. $9x^2 + 4y^2 = 36$ **ellipse** $\frac{x^2}{4} + \frac{y^2}{9} = 1$



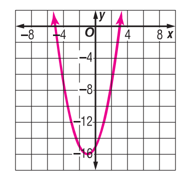
3. $x^2 + y^2 - 16 = 0$ **circle** $x^2 + y^2 = 16$



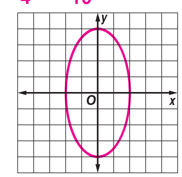
4. $x^2 + 8x + y^2 = 9$ **circle** $(x + 4)^2 + y^2 = 25$



5. $x^2 + 2x - 15 = y$ **parabola** $y = (x + 1)^2 - 16$



6. $100x^2 + 25y^2 = 400$ **ellipse** $\frac{x^2}{4} + \frac{y^2}{16} = 1$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $9x^2 + 4y^2 = 36$ **ellipse**
8. $x^2 + y^2 = 25$ **circle**
9. $y = x^2 + 2x$ **parabola**
10. $y = 2x^2 - 4x - 4$ **parabola**
11. $4y^2 - 25x^2 = 100$ **hyperbola**
12. $16x^2 + y^2 = 16$ **ellipse**
13. $16x^2 - 4y^2 = 64$ **hyperbola**
14. $5x^2 + 5y^2 = 25$ **circle**
15. $25y^2 + 9x^2 = 225$ **ellipse**
16. $36y^2 - 4x^2 = 144$ **hyperbola**
17. $y = 4x^2 - 36x - 144$ **parabola**
18. $x^2 + y^2 - 144 = 0$ **circle**
19. $(x + 3)^2 + (y - 1)^2 = 4$ **circle**
20. $25y^2 - 50y + 4x^2 = 75$ **ellipse**
21. $x^2 - 6y^2 + 9 = 0$ **hyperbola**
22. $x = y^2 + 5y - 6$ **parabola**
23. $(x + 5)^2 + y^2 = 10$ **circle**
24. $25x^2 + 10y^2 - 250 = 0$ **ellipse**

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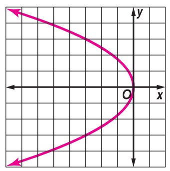
10-6 Practice

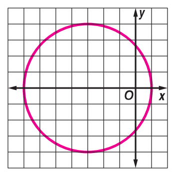
Conic Sections

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

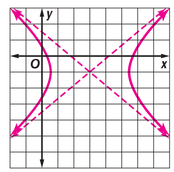
1. $y^2 = -3x$
parabola
$x = -\frac{1}{3}y^2$



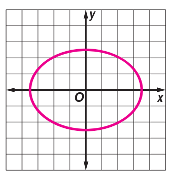
2. $x^2 + y^2 + 6x = 7$
circle
$(x + 3)^2 + y^2 = 16$



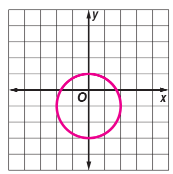
3. $5x^2 - 6y^2 - 30x - 12y = -9$
hyperbola
$\frac{(x-3)^2}{6} - \frac{(y+1)^2}{5} = 1$



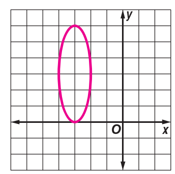
4. $196y^2 = 1225 - 100x^2$
ellipse
$\frac{x^2}{12.25} + \frac{y^2}{6.25} = 1$



5. $3x^2 = 9 - 3y^2 - 6y$
circle
$x^2 + (y + 1)^2 = 4$



6. $9x^2 + y^2 + 54x - 6y = -81$
ellipse
$\frac{(x+3)^2}{1} + \frac{(y-3)^2}{9} = 1$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $6x^2 + 6y^2 = 36$
circle

8. $4x^2 - y^2 = 16$
hyperbola

9. $9x^2 + 16y^2 - 64y - 80 = 0$
ellipse

10. $5x^2 + 5y^2 - 45 = 0$
circle

11. $x^2 + 2x = y$
parabola

12. $4y^2 - 36x^2 + 4x - 144 = 0$
hyperbola

13. ASTRONOMY A satellite travels in an hyperbolic orbit. It reaches the vertex of its orbit at (5, 0) and then travels along a path that gets closer and closer to the line $y = \frac{2}{5}x$. Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at (0, 0).
$\frac{x^2}{25} - \frac{y^2}{4} = 1$

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10-6 Word Problem Practice

Conic Sections

1. MISSING INFORMATION Rick began reading a book on conic sections. He came to this passage and discovered an inkblot covering part of an equation.

> For example, although it may not be obvious, the equation below describes a circle.
> $7x^2 - 12x + \;\; y^2 - 16y - 94 = 0$
> To see that it is a circle, observe that the

Based on the information in the passage and your own knowledge of conic sections, what number is being covered by the inkblot?
7

2. HEADLIGHTS The light from the headlight of a car is in the shape of a cone. The axis of the cone is parallel to the ground. What shape does the edge of the lit region form on the road, assuming that the road is flat and level?
hyperbola

3. REASONING Jason has been struggling with conic sections. He decides he needs more practice, but he needs to have a way of making practice equations. He decides to use an equation of the form

$$Ax^2 + By^2 = 1,$$

where A and B are determined by rolling a pair of dice. After several rolls, he begins to realize that this system is not good enough because some conic sections never appear. Which types of conic section cannot occur using his method?
parabolas and hyperbolas are not possible

4. MIRROR A painter used a can of spray paint to make an image. The boundary of the image is described by the equation

$$4x^2 - 16x + y^2 - 6y + 21 = 0.$$

Put this equation into standard form and describe whether the curve is a circle, ellipse, parabola, or hyperbola.
$(x - 2)^2 + \frac{(y-3)^2}{4} = 1$, **ellipse**

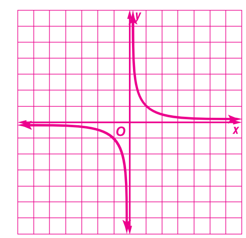
NONSTANDARD EQUATIONS

For Exercises 5–7, use the following information.

Consider the equation $xy = 1$.

5. Are there any solutions of this equation that lie on the x- or y-axis?
no

6. Sketch a graph of the solutions of the equation.



7. Assuming that the equation represents a conic section, based on the graph, which type of conic section is it?
a hyperbola

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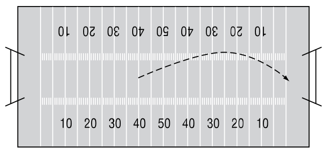
10-6 Enrichment

Parabolic Football

A parabola is defined as all the points (x, y) in the plane whose distance from a fixed point, called the **focus** is the same as its distance from a fixed line, called the **directrix**. Examples of parabolas are the cables on a suspension bridge, satellite dishes, and the flight path of a football during a kick-off.



At the kick-off at the beginning of a football game the ball is placed on the 40-yard line of the kicking team. Suppose the receiving team catches the ball on the goal line. Assume the 50-yard line has coordinates (0, 0) and the 40-yard line of the kicking team has coordinates (−10, 0).

1. Determine which equation of the parabola best describes this situation. For the other choices explain why they do not make sense to the situation.

a. $y = 50x(x + 10)$ **Incorrect. This says the goal line is the 50-yard line**

b. $y = 100(x - 50)(x + 10)$ **Incorrect. Wrong direction and way too high.**

c. $y = -\frac{1}{4}(x - 50)(x + 10)$ **Correct**

d. $y = -20(x - 50)(x - 60)$ **Incorrect. Too high, not right yard lines.**

2. During the same game, the quarterback throws a forward pass from the 50-yard line to his receiver on the 25-yard line. Assuming the ball follows the path of a parabola, write an equation modeling the flight path of the ball from quarterback to receiver.
Answer may vary based on orientation.

3. The team did not pick up the first down, so they elect to try a field goal. Fortunately, one of the assistant coaches is a part-time mathematician and found an equation that describes their kicker as: $y = -0.2x^2 + 8x$. The line of scrimmage is the 25-yard line, so the ball will be placed on the 32-yard line for the kick. Add 10 more yards for the depth of the end zone (goal line to the goal post), making it a 42-yard field goal attempt. Will your kicker make it?
Setting the vertex at the origin, then the focus is located at $\left(0, \frac{1}{8}\right)$.

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10-7 Lesson Reading Guide

Solving Quadratic Systems

Get Ready for the Lesson

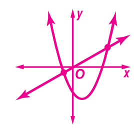
Read the introduction to Lesson 10-7 in your textbook.

The figure in your textbook shows that the spaceship hits the circular force field in two points. Is it possible for the spaceship to hit the force field in either fewer or more than two points? State all possibilities and explain how these could happen. **Sample answer: The spaceship could hit the force field in zero points if the spaceship missed the force field all together. The spaceship could also hit the force field in one point if the spaceship just touched the edge of the force field.**

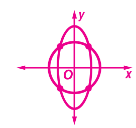
Read the Lesson

1. Draw a sketch to illustrate each of the following possibilities.

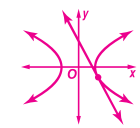
a. a parabola and a line that intersect in 2 points



b. an ellipse and a circle that intersect in 4 points



c. a hyperbola and a line that intersect in 1 point



2. Consider the following system of equations.

$x^2 = 3 + y^2$
$2x^2 + 3y^2 = 11$

a. What kind of conic section is the graph of the first equation? **hyperbola**

b. What kind of conic section is the graph of the second equation? **ellipse**

c. Based on your answers to parts a and b, what are the possible numbers of solutions that this system could have? **0, 1, 2, 3, or 4**

Remember What You Learned

3. Suppose that the graph of a quadratic inequality is a region whose boundary is a circle. How can you remember whether to shade the interior or the exterior of the circle?
Sample answer: The solutions of an inequality of the form $x^2 + y^2 < r^2$ are all points that are *less than r* units from the origin, so the graph is the *interior* of the circle. The solutions of an inequality of the form $x^2 + y^2 > r^2$ are the points that are *more than r* units from the origin, so the graph is the *exterior* of the circle.

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10-7 Study Guide and Intervention

Solving Quadratic Systems

Systems of Quadratic Equations Like systems of linear equations, systems of quadratic equations can be solved by substitution and elimination. If the graphs are a conic section and a line, the system will have 0, 1, or 2 solutions. If the graphs are two conic sections, the system will have 0, 1, 2, 3, or 4 solutions.

Example **Solve the system of equations.** $y = x^2 - 2x - 15$
$x + y = -3$

Rewrite the second equation as $y = -x - 3$ and substitute into the first equation.

$-x - 3 = x^2 - 2x - 15$

$0 = x^2 - x - 12$ Add $x + 3$ to each side.

$0 = (x - 4)(x + 3)$ Factor.

Use the Zero Product property to get

$x = 4$ or $x = -3$.

Substitute these values for x in $x + y = -3$:

$4 + y = -3$ or $-3 + y = -3$

$y = -7$ $y = 0$

The solutions are $(4, -7)$ and $(-3, 0)$.

Exercises

Find the exact solution(s) of each system of equations.

1. $y = x^2 - 5$
$y = x - 3$
$(2, -1), (-1, -4)$

2. $x^2 + (y - 5)^2 = 25$
$y = -x^2$
$(0, 0)$

3. $x^2 + (y - 5)^2 = 25$
$y = x^2$
$(0, 0), (3, 9), (-3, 9)$

4. $x^2 + y^2 = 9$
$x^2 + y = 3$
$(0, 3), (\sqrt{5}, -2), (-\sqrt{5}, -2)$

5. $x^2 - y^2 = 1$
$x^2 + y^2 = 16$
$\left(\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right)$

6. $y = x - 3$
$x = y^2 - 4$
$\left(\frac{7 + \sqrt{29}}{2}, \frac{1 + \sqrt{29}}{2}\right), \left(\frac{7 - \sqrt{29}}{2}, \frac{1 - \sqrt{29}}{2}\right)$

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10-7 Study Guide and Intervention *(continued)*

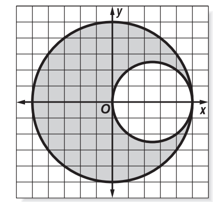
Solving Quadratic Systems

Systems of Quadratic Inequalities Systems of quadratic inequalities can be solved by graphing.

Example 1 **Solve the system of inequalities by graphing.**

$x^2 + y^2 \le 25$

$\left(x - \frac{5}{2}\right)^2 + y^2 \ge \frac{25}{4}$

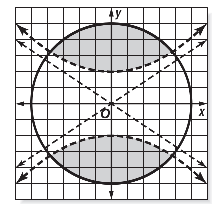


The graph of $x^2 + y^2 \le 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\left(x - \frac{5}{2}\right)^2 + y^2 \ge \frac{25}{4}$ consists of all points on or outside the circle with center $\left(\frac{5}{2}, 0\right)$ and radius $\frac{5}{2}$. The solution of the system is the set of points in both regions.

Example 2 **Solve the system of inequalities by graphing.**

$x^2 + y^2 \le 25$

$\frac{y^2}{4} - \frac{x^2}{9} > 1$

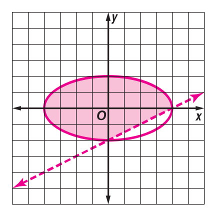


The graph of $x^2 + y^2 \le 25$ consists of all points on or inside the circle with center (0, 0) and radius 5.The graph of $\frac{y^2}{4} - \frac{x^2}{9} > 1$ are the points "inside" but not on the branches of the hyperbola shown. The solution of the system is the set of points in both regions.

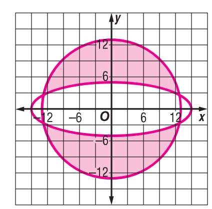
Exercises

Solve each system of inequalities below by graphing.

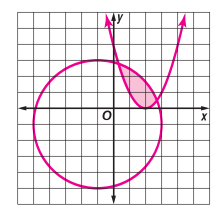
1. $\frac{x^2}{16} + \frac{y^2}{4} \le 1$
$y > \frac{1}{2}x - 2$



2. $x^2 + y^2 \le 169$
$x^2 + 9y^2 \ge 225$



3. $y \ge (x - 2)^2$
$(x + 1)^2 + (y + 1)^2 \le 16$



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10-7 Skills Practice

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $y = x - 2$, $y = x^2 - 2$ **(0, −2), (1, −1)**

2. $y = x + 3$, $y = 2x^2$ **(−1, 2), (1.5, 4.5)**

3. $y = 3x$, $x = y^2$ **$(0, 0), \left(\frac{1}{9}, \frac{1}{3}\right)$**

4. $y = x$, $x^2 + y^2 = 4$ **$(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})$**

5. $x = -5$, $x^2 + y^2 = 25$ **(−5, 0)**

6. $y = 7$, $x^2 + y^2 = 9$ **no solution**

7. $y = -2x + 2$, $y^2 = 2x$ **$(2, -2), \left(\frac{1}{2}, 1\right)$**

8. $x - y + 1 = 0$, $y^2 = 4x$ **(1, 2)**

9. $y = 2 - x$, $y = x^2 - 4x + 2$ **(0, 2), (3, −1)**

10. $y = x - 1$, $y = x^2$ **no solution**

11. $y = 3x^2$, $y = -3x^2$ **(0, 0)**

12. $y = x^2 + 1$, $y = -x^2 + 3$ **(−1, 2), (1, 2)**

13. $y = 4x$, $4x^2 + y^2 = 20$ **(−1, −4), (1, 4)**

14. $y = -1$, $4x^2 + y^2 = 1$ **(0, −1)**

15. $4x^2 + 9y^2 = 36$, $x^2 - 9y^2 = 9$ **(−3, 0), (3, 0)**

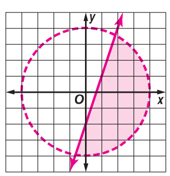
16. $3(y + 2)^2 - 4(x - 3)^2 = 12$, $y = -2x + 2$ **(0, 2), (3, −4)**

17. $x^2 - 4y^2 = 4$, $x^2 + y^2 = 4$ **(−2, 0), (2, 0)**

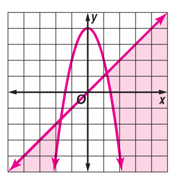
18. $y^2 - 4x^2 = 4$, $y = 2x$ **no solution**

Solve each system of inequalities by graphing.

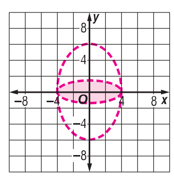
19. $y \le 3x - 2$, $x^2 + y^2 < 16$



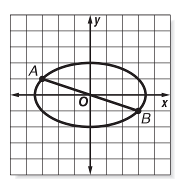
20. $y \le x$, $y \ge -2x^2 + 4$



21. $4y^2 + 9x^2 < 144$, $x^2 + 8y^2 < 16$



22. GARDENING An elliptical garden bed has a path from point A to point B. If the bed can be modeled by the equation $x^2 + 3y^2 = 12$ and the path can be modeled by the line $y = -\frac{1}{3}x$, what are the coordinates of points A and B? **(−3, 1) and (3, −1)**



NAME ______________________ DATE ____________ PERIOD _____

10-7 Practice

Solving Quadratic Systems

Find the exact solution(s) of each system of equations.

1. $(x - 2)^2 + y^2 = 5$, $x - y = 1$ **(0, −1), (3, 2)**

2. $x = 2(y + 1)^2 - 6$, $x + y = 3$ **(2, 1), (6.5, −3.5)**

3. $y^2 - 3x^2 = 6$, $y = 2x - 1$ **(−1, −3), (5, 9)**

4. $x^2 + 2y^2 = 1$, $y = -x + 1$ **$(1, 0), \left(\frac{1}{3}, \frac{2}{3}\right)$**

5. $4y^2 - 9x^2 = 36$, $4x^2 - 9y^2 = 36$ **no solution**

6. $y = x^2 - 3$, $x^2 + y^2 = 9$ **$(0, -3), (\pm\sqrt{5}, 2)$**

7. $x^2 + y^2 = 25$, $4y = 3x$ **(4, 3), (−4, −3)**

8. $y^2 = 10 - 6x^2$, $4y^2 = 40 - 2x^2$ **$(0, \pm\sqrt{10})$**

9. $x^2 + y^2 = 25$, $x = 3y - 5$ **(−5, 0), (4, 3)**

10. $4x^2 + 9y^2 = 36$, $2x^2 - 9y^2 = 18$ **(±3, 0)**

11. $x = -(y - 3)^2 + 2$, $x = (y - 3)^2 + 3$ **no solution**

12. $\frac{x^2}{9} - \frac{y^2}{16} = 1$, $x^2 + y^2 = 9$ **(±3, 0)**

13. $25x^2 + 4y^2 = 100$, $x = -\frac{5}{2}$ **no solution**

14. $x^2 + y^2 = 4$, $\frac{x^2}{4} + \frac{y^2}{8} = 1$ **(±2, 0)**

15. $x^2 - y^2 = 3$, $y^2 - x^2 = 3$ **no solution**

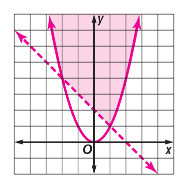
16. $\frac{x^2}{7} + \frac{y^2}{7} = 1$, $3x^2 - y^2 = 9$ **$(\pm 2, \pm\sqrt{3})$**

17. $x + 2y = 3$, $x^2 + y^2 = 9$ **$(3, 0), \left(-\frac{9}{5}, \frac{12}{5}\right)$**

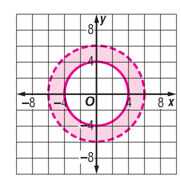
18. $x^2 + y^2 = 64$, $x^2 - y^2 = 8$ **$(\pm 6, \pm 2\sqrt{7})$**

Solve each system of inequalities by graphing.

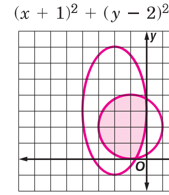
19. $y \ge x^2$, $y > -x + 2$



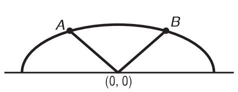
20. $x^2 + y^2 < 36$, $x^2 + y^2 \ge 16$



21. $\frac{(y - 3)^2}{16} + \frac{(x + 2)^2}{4} \le 1$, $(x + 1)^2 + (y - 2)^2 \le 4$



22. GEOMETRY The top of an iron gate is shaped like half an ellipse with two congruent segments from the center of the ellipse to the ellipse as shown. Assume that the center of the ellipse is at (0, 0). If the ellipse can be modeled by the equation $x^2 + 4y^2 = 4$ for $y \ge 0$ and the two congruent segments can be modeled by $y = \frac{\sqrt{3}}{2}x$ and $y = -\frac{\sqrt{3}}{2}x$, what are the coordinates of points A and B?



$\left(-1, \frac{\sqrt{3}}{2}\right)$ and $\left(1, \frac{\sqrt{3}}{2}\right)$