

8-1

Skills Practice

Key

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{21x^3y}{14x^2y^2} \cdot \frac{3x}{2y}$

2. $\frac{5ab^3}{25a^2b^2} \cdot \frac{b}{5a}$

3. $\frac{(x^6)^3}{(x^3)^4} \cdot X^6$

4. $\frac{8y^2(y^6)^3}{4y^{24}} \cdot \frac{2}{y^4}$

5. $\frac{18}{2x-6} \cdot \frac{9}{x-3}$

6. $\frac{x^2-4}{(x-2)(x+1)} \cdot \frac{x+2}{x+1}$

7. $\frac{3a^2-24a}{3a^2+12a} \cdot \frac{a-8}{a+4}$

8. $\frac{3m}{2n} \cdot \frac{n^3}{6} \cdot \frac{mn^2}{4}$

9. $\frac{24e^3}{5f^2} \cdot \frac{10(ef)^3}{8e^5f} \cdot 6e$

10. $\frac{5s^2}{s^2-4} \cdot \frac{s+2}{10s^5} \cdot \frac{1}{25^3(s-2)}$

11. $\frac{7g}{y^2} \div 21g^3 \cdot \frac{1}{3g^2y^2}$

12. $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5} \cdot \frac{32z^7}{35v^2y}$

13. $\frac{3x^2}{x+2} \div \frac{3x}{x^2-4} \cdot X(X-2)$

14. $\frac{q^2+2q}{6q} \div \frac{q^2-4}{3q^2} \cdot \frac{q^2}{2(q-2)}$

15. $\frac{w^2-5w-24}{w+1} \cdot \frac{w^2-6w-7}{w+3} \cdot (w-8)(w-7)$

16. $\frac{t^2+19t+84}{4t-4} \cdot \frac{2t-2}{t^2+9t+14} \cdot \frac{t+12}{2(t+2)}$

17. $\frac{x^2-5x+4}{2x-8} \div (3x^2-3x) \cdot \frac{1}{6x}$

18. $\frac{16a^2+40a+25}{3a^2-10a-8} \div \frac{4a+5}{a^2-8a+16} \cdot \frac{(4a+5)(a-4)}{3a+2}$

19. $\frac{\frac{c^2}{2d^2}}{-\frac{c^6}{5d}} \cdot \frac{-5}{2c^4d}$

20. $\frac{\frac{a^2-b^2}{4a}}{\frac{a+b}{2a}} \cdot \frac{a-b}{2}$

8-1 Practice

Multiplying and Dividing Rational Expressions

Key

Simplify each expression.

1. $\frac{9a^2b^3}{27a^4b^4c}$

$\frac{1}{3a^2bc}$

2. $\frac{(2m^3n^2)^3}{-18m^5n^4}$

$-\frac{4m^9n^6}{9}$

3. $\frac{10y^2 + 15y}{35y^2 - 5y}$

$\frac{2y+3}{7y-1}$

4. $\frac{2k^2 - k - 15}{k^2 - 9}$

$\frac{2k+5}{k+3}$

5. $\frac{25 - v^2}{3v^2 - 13v - 10}$

$-\frac{v+5}{3v+2}$

6. $\frac{x^4 + x^3 - 2x^2}{x^4 - x^3}$

$\frac{x+2}{x}$

7. $\frac{-2u^3y}{15xz^5} \cdot \frac{25x^3}{14u^2y^2}$

$-\frac{5ux^2}{21yz^5}$

8. $\frac{a+y}{6} \cdot \frac{4}{y+a}$

$\frac{2}{3}$

9. $\frac{n^5}{n-6} \cdot \frac{n^2-6n}{n^8}$

$\frac{1}{n^2}$

10. $\frac{a-y}{w+n} \cdot \frac{w^2-n^2}{y-a}$

$w-w$

11. $\frac{x^2 - 5x - 24}{6x + 2x^2} \cdot \frac{5x^2}{8-x}$

$-\frac{5x}{2}$

12. $\frac{x-5}{10x-2} \cdot \frac{25x^2-1}{x^2-10x+25}$

$\frac{5x+1}{2(x-5)}$

13. $\frac{a^5y^3}{wy^7} \div \frac{a^3w^2}{w^5y^2}$

$\frac{a^2w^2}{y^2}$

14. $\left(\frac{2xy}{w^2}\right)^3 \div \frac{24x^2}{w^5}$

$\frac{xy^3}{3w}$

15. $\frac{x+y}{6} \div \frac{x^2-y^2}{3}$

$\frac{1}{2(x-y)}$

16. $\frac{3x+6}{x^2-9} \div \frac{6x^2+12x}{4x+12}$

$\frac{2}{x(x-3)}$

17. $\frac{2s^2-7s-15}{(s+4)^2} \div \frac{s^2-10s+25}{s+4}$

$\frac{2s+3}{(s+4)(s-5)}$

18. $\frac{9-a^2}{a^2+5a+6} \div \frac{2a-6}{5a+10}$

$-\frac{5}{2}$

19. $\frac{2x+1}{\frac{x}{4-x}}$

$\frac{2x+1}{4-x}$

20. $\frac{\frac{x^2-9}{4}}{3-x}$

$-\frac{2(x+3)}{3-x}$

21. $\frac{\frac{x^3+2^3}{x^2-2x}}{(x+2)^3}$

$\frac{x^2-2x+4}{x(x-2)}$

22. **GEOMETRY** A right triangle with an area of $x^2 - 4$ square units has a leg that measures $2x + 4$ units. Determine the length of the other leg of the triangle.

$x-2$

$A = \frac{1}{2}bh$

23. **GEOMETRY** A rectangular pyramid has a base area of $\frac{x^2 + 3x - 10}{2x}$ square centimeters

and a height of $\frac{x^2 - 3x}{x^2 - 5x + 6}$ centimeters. Write a rational expression to describe the

volume of the rectangular pyramid.

$V = \frac{1}{3}(\text{Area base})(\text{height})$

$\frac{x+5}{6} \text{ cm}^3$

8-2

Skills Practice

Key

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $12c, 6c^2d$

$12c^2d$

2. $18a^3bc^2, 24b^2c^2$

$72a^3b^2c^2$

3. $2x - 6, x - 3$

$2(x-3)$

4. $5a, a - 1$

$5a(a-1)$

5. $t^2 - 25, t + 5$

$(t+5)(t-5)$

6. $x^2 - 3x - 4, x + 1$

$(x-4)(x+1)$

Simplify each expression.

7. $\frac{3}{x} + \frac{5}{y}$

$\frac{5x+3y}{xy}$

8. $\frac{3}{8p^2q} + \frac{5}{4p^2q}$

$\frac{13}{8p^2q}$

9. $\frac{2c-7}{3} + 4$

$\frac{2c+5}{3}$

10. $\frac{2}{m^2n} + \frac{5}{n}$

$\frac{2+5m^2}{m^2n}$

11. $\frac{12}{5y^2} - \frac{2}{5yz}$

$\frac{12z-2y}{5y^2z}$

12. $\frac{7}{4gh} + \frac{3}{4h^2}$

$\frac{7h+3g}{4gh^2}$

13. $\frac{2}{a+2} - \frac{3}{2a}$

$\frac{a-6}{2a(a+2)}$

14. $\frac{5}{3b+d} - \frac{2}{3bd}$

$\frac{15bd-6b-2d}{3bd(3b+d)}$

15. $\frac{3}{w-3} - \frac{2}{w^2-9}$

$\frac{3w+7}{(w-3)(w+3)}$

16. $\frac{3t}{2-x} + \frac{5}{x-2}$

$\frac{5-3t}{x-2}$

17. $\frac{m}{m-n} - \frac{m}{n-m}$

$\frac{2m}{m-n}$

18. $\frac{4z}{z-4} + \frac{z+4}{z+1}$

$\frac{5z^2+4z-16}{(z-4)(z+1)}$

19. $\frac{1}{x^2+2x+1} + \frac{x}{x+1}$

$\frac{x^2+x+1}{(x+1)^2}$

20. $\frac{2x+1}{x-5} - \frac{4}{x^2-3x-10}$

$\frac{2x^2+5x-2}{(x-5)(x+2)}$

21. $\frac{n}{n-3} + \frac{2n+2}{n^2-2n-3}$

$\frac{n+2}{n-3}$

22. $\frac{3}{y^2+y-12} - \frac{2}{y^2+6y+8}$

$\frac{y+12}{(y+4)(y-3)(y+2)}$

8-2 Practice**Adding and Subtracting Rational Expressions***Key*

Find the LCM of each set of polynomials.

1. x^2y, xy^3

$$x^2y^3$$

2. a^2b^3c, abc^4

$$a^2b^3c^4$$

3. $x + 1, x + 3$

$$(x+1)(x+3)$$

4. $g - 1, g^2 + 3g - 4$

$$(g-1)(g+4)$$

5. $2r + 2, r^2 + r, r + 1$

$$2r(r+1)$$

6. $3, 4w + 2, 4w^2 - 1$

$$6(2w+1)(2w-1)$$

7. $x^2 + 2x - 8, x + 4$

$$(x-2)(x+4)$$

8. $x^2 - x - 6, x^2 + 6x + 8$

$$(x+2)(x+4)(x-3)$$

9. $d^2 + 6d + 9, 2(d^2 - 9)$

$$2(d-3)(d+3)^2$$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$

$$\frac{20-21b}{24ab}$$

11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$

$$\frac{25y^2-12x^2}{60x^4y^3}$$

12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$

$$\frac{2d^2+9c}{12c^2d^3}$$

13. $\frac{4m}{3mn} + 2$

$$\frac{2(2+3n)}{3n}$$

14. $2x - 5 \frac{x-8}{x+4}$

$$\frac{2(x+3)(x-2)}{x+4}$$

15. $\frac{1}{a-3} + \frac{9}{a-5}$

$$\frac{13a-47}{(a-3)(a-5)}$$

16. $\frac{16}{x^2-16} + \frac{2}{x+4}$

$$\frac{2}{x-4}$$

17. $\frac{2-5m}{m-9} + \frac{4m-5}{9-m}$

$$\frac{7-9m}{m-9}$$

18. $\frac{y}{y^2-3y-10} + \frac{y}{y^2+y-2}$

$$\frac{2y-1}{(y+2)(y-1)}$$

19. $\frac{5}{2x-12} - \frac{20}{x^2-4x-12}$

$$\frac{5}{2(x+2)}$$

20. $\frac{2p-3}{p^2-5p+6} - \frac{5}{p^2-9}$

$$\frac{2p^2-2p+1}{(p-2)(p+3)(p-3)}$$

21. $\frac{2}{5n-4} + \frac{7}{10n}$

$$\frac{2(6-5n)}{20n}$$

22. $\frac{2a}{a-3} - \frac{2a}{a+3} + \frac{36}{a^2-9}$

$$\frac{12}{a-3}$$

23. $\frac{1}{x-y} + \frac{1}{x+y}$

$$\frac{3x+y}{x+y}$$

24. $\frac{r+6}{r} - \frac{1}{r+2}$

$$\frac{r+4}{r+1}$$

25. **GEOMETRY** The expressions $\frac{5x}{2}$, $\frac{20}{x+4}$, and $\frac{10}{x-4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle.

$$\frac{5(x^3-4x-16)}{2(x-4)(x+4)}$$

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip.

$$\frac{2}{r+2} + \frac{2}{r-2}$$

$$\frac{4r}{(r+2)(r-2)} h$$

Very

8-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{3}{x^2 - 2x - 8}$

ver. as.
 $x = 4$
 $x = -2$

3. $f(x) = \frac{x + 12}{x^2 + 10x - 24}$

v. asym. $x = 2$
hole: $x = -12$

5. $f(x) = \frac{x^2 + 8x + 12}{x + 2}$

hole: $x = -2$

2. $f(x) = \frac{10}{x^2 - 13x + 36}$

v. asym. $x = 4$
 $x = 9$

4. $f(x) = \frac{x - 1}{x^2 - 4x + 3}$

v. as. $x = 3$
hole: $x = 1$

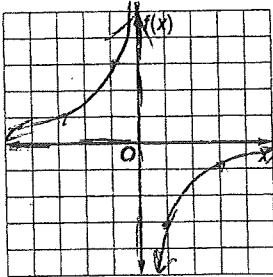
6. $f(x) = \frac{x^2 + x - 12}{x - 3}$

hole: $x = 3$

Graph each rational function.

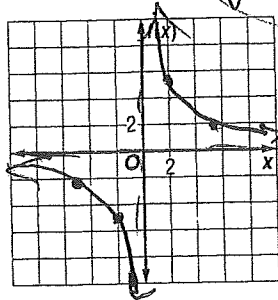
7. $f(x) = \frac{-3}{x}$

ver. t. $x = 0$
hor. as. $y = 0$

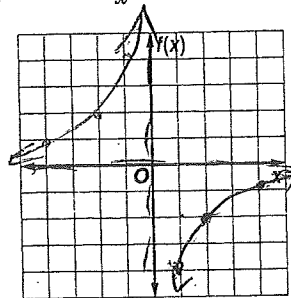


8. $f(x) = \frac{10}{x}$

ver. as. $x = 0$
hor. as. $y = 0$



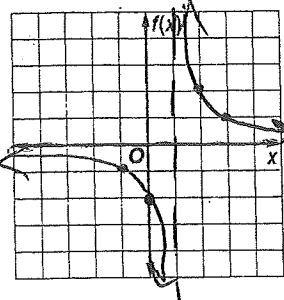
9. $f(x) = \frac{-4}{x}$



x	y
1	-4
2	-2
4	-1

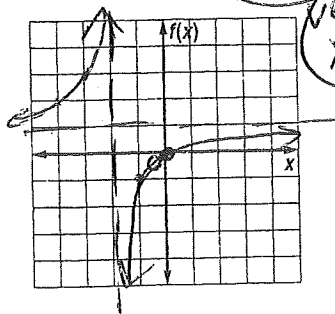
10. $f(x) = \frac{2}{x - 1}$

ver. as. $x = 1$
hor. as. $y = 0$



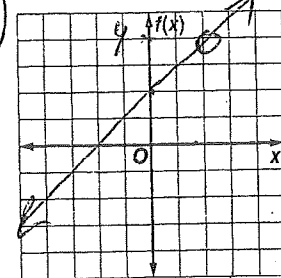
11. $f(x) = \frac{1 \cdot x}{x + 2}$

hor. as. $y = 1$



12. $f(x) = \frac{x^2 - 4}{x - 2} = x + 2$

ver. as. $x = 2$



$x = 2$ a hole

$f(x) = \frac{(x-2)(x+2)}{x-2}$

D: $(-\infty, 2) \cup (2, \infty)$

R: $(-\infty, 4) \cup (4, \infty)$

8-3 Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$

v. as. $x=2, x=-5$

2. $f(x) = \frac{x-7}{x^2 - 10x + 21}$

v. as. $x=3$
hole: $x=7$

3. $f(x) = \frac{x-2}{x^2 + 4x + 4}$

v. as. $x=-2$

4. $f(x) = \frac{x^2 - 100}{x + 10}$

hole: $x=-10$

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$

hole: $x=6$

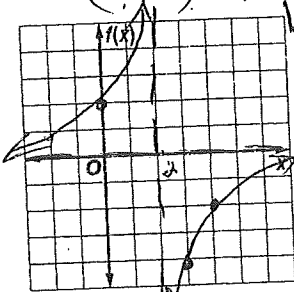
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$

hole $x=-5$

Graph each rational function.

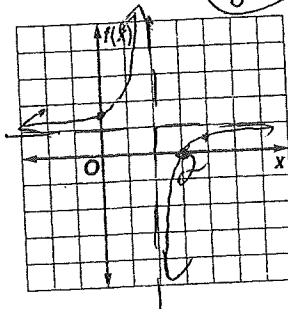
7. $f(x) = \frac{-4}{x-2}$

vert. as. $x=2$
h. as. $y=0$



8. $f(x) = \frac{x-3}{x-2}$

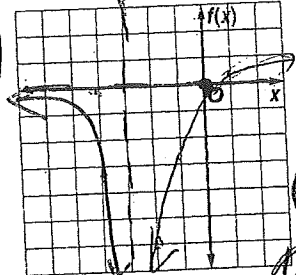
h. as. $y=1$
vert. as. $x=2$



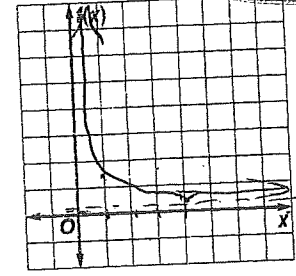
9. $f(x) = \frac{3x}{(x+3)^2}$

x	y
0	0
1	3/16
-4	-1/2

vert. as. $x=-3$
hor. as. $y=0$

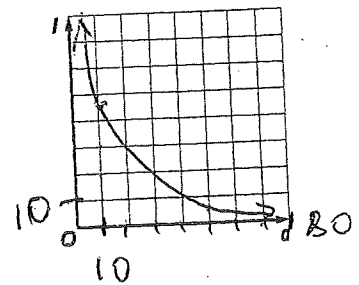


10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6+x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6+x}{6x}$ for $x > 0, y > 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour? What domain and range values are meaningful in the context of the problem?



$f(4) = \frac{10}{24} = \frac{5}{12}$. The number of hours it takes her father to give the shed a coat of paint should be positive. Therefore, $x > 0, f(x) > \frac{1}{6}$

11. **LIGHT** The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 < I \leq 80$ and $0 < d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source? What domain and range values are meaningful in the context of the problem?



$I(20) = \frac{4500}{400} = 11.25$ foot-candles. The distance of the object from the source should be positive. Therefore, $d > 0$ & $I > 0$.

Review

Name: _____ Class: _____ Date: _____

ID: A

Key

Chapter 8, algebra 2 (8.1-8.3)

Simplify the given expression.

$$1. \frac{\cancel{7}p^4}{5q^5(r-5)^3} \cdot \frac{37q^2(r-5)}{\cancel{30}p^2} = \frac{37p}{50q^3(r-5)^2}$$

$$2. \frac{\cancel{12}x}{2y} \cdot \frac{3y^2}{\cancel{24}x^3 \cdot 2} = \frac{3y}{4x^2}$$

$$3. \frac{5x^2y^3}{3a^5b^4} \div \frac{23x^5y}{42a^7b^3} = \frac{\cancel{5}x^2y^3}{\cancel{3}a^5b^4} \cdot \frac{4\cancel{2}a^7b^3}{23x^{\cancel{5}}y} = \frac{70y^2a^2}{23x^3b}$$

$$4. \frac{5(a^2+5a+6)}{3(a^2-36)} \div \frac{41(a+3)}{6(a+6)} = \frac{\cancel{5}(a+\cancel{3})(a+2)}{\cancel{3}(a-6)(a+6)} \cdot \frac{\cancel{6}^2(a+6)}{41(a+\cancel{3})} = \frac{10(a+2)}{41(a-6)}$$

$$5. \frac{\frac{d}{5f^2}}{\frac{d^4}{32f^3}} =$$

$$\frac{32d^1 f^3}{5f^2 d^4} = \frac{32f}{5d^3}$$

$$6. \frac{\frac{8(x^2-8)}{x}}{\frac{8(x^2+8)}{26x^2-31x}} =$$

$$\frac{\cancel{8}(x-1)(x+1) \cdot \cancel{x}(26x-31)}{\cancel{x} \cdot \cancel{8}(x^2+8)} =$$

$$\frac{(x-1)(x+1)(26x-31)}{x^2+8}$$

Find the LCM of the set of polynomials.

$$7. (121x^2 - 9y^2)(11x^2 + 3yx)$$

$$(11x-3y)(11x+3y); x(11x+3y)$$

$$\text{LCM: } (x(11x+3y))(11x-3y)$$

$$8. 29a^3c, 16b^4, b^2c^2$$

$$29 \cdot 16 \cdot a^3 b^4 c^2 = (464a^3b^4c^2)$$

Simplify the given expression.

$$9. \frac{3}{4x^2-25} + \frac{2}{2x+5} =$$

$$\frac{3}{(2x-5)(2x+5)} + \frac{2}{2x+5} =$$

$$\frac{3+4x-10}{(2x-5)(2x+5)}$$

$$= \frac{4x-7}{(2x-5)(2x+5)}$$

$$10. \frac{17 + \frac{2a}{11b}}{1} =$$

$$\frac{187b + 2a}{11b}$$

$$\frac{17(11b)}{11b} + \frac{2a}{11b}$$

11. $\frac{19}{xy^2} - \frac{7y^2}{8x^2y^3}$

$\frac{152x - 7y^4}{8x^2y^2}$

$x \neq 0, y \neq 0$

12. $\frac{8}{y+2} - \frac{3y}{y^2-4}$

$\frac{8(y-2) - 3y}{(y+2)(y-2)} = \frac{8y-16-3y}{(y-2)(y+2)} = \frac{5y-16}{(y-2)(y+2)}$

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of the rational function.

13. $f(x) = \frac{3}{x^2 - 14x + 48} = \frac{3x^0}{(x-6)(x-8)}$

HA: $y=0$
 v. asymp.: $x=6$
 $x=8$

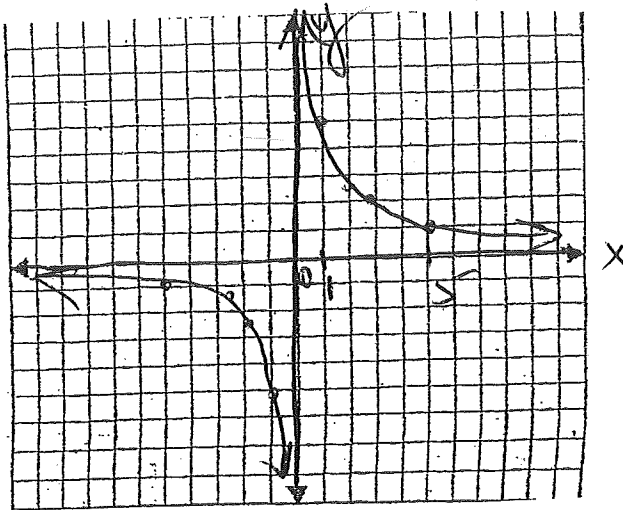
14. $f(x) = \frac{x-6}{x^2 - 7x + 6} = \frac{\cancel{x-6}}{(\cancel{x-6})(x-1)}$

v. asymp. $x=1$
 hole: $x=6$
 HA $y=0$

Graph the rational function.

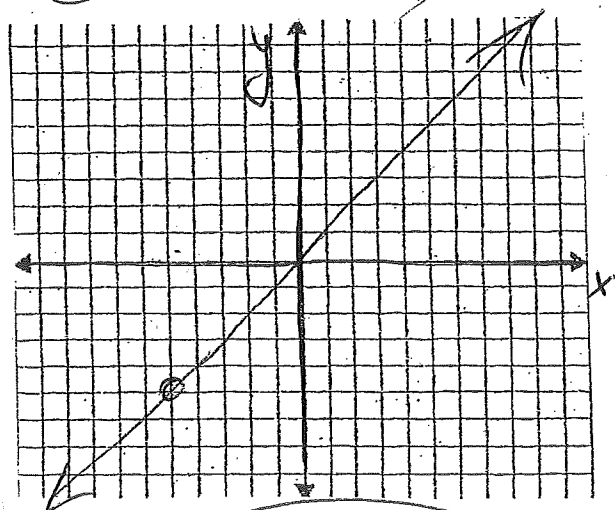
✓ 15. $f(x) = \frac{5}{x}$

vert. as. $x=0$
hor. as. $y=0$



x	y
1	5
2	2.5
5	1
-1	-5

✓ (17.) $f(x) = \frac{x^2 + 5x}{x+5} = \frac{x(x+5)}{x+5} = x$

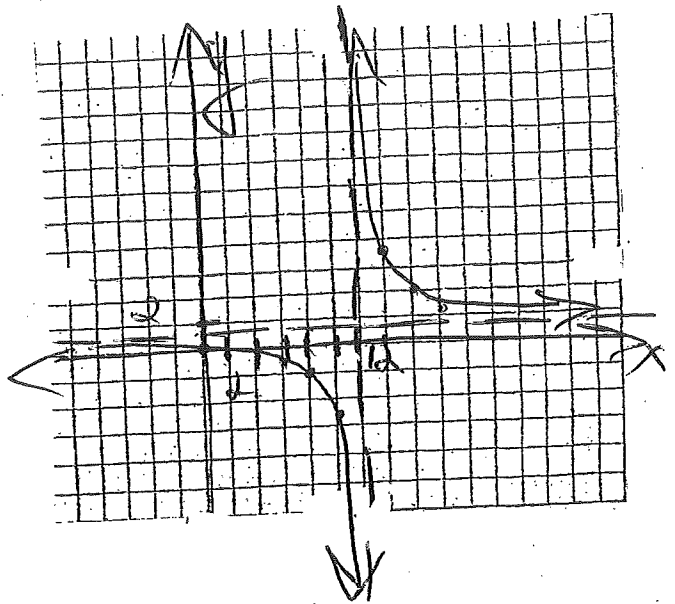


(hole: $x = -5$)

✓ 16. $f(x) = \frac{x}{x-12}$

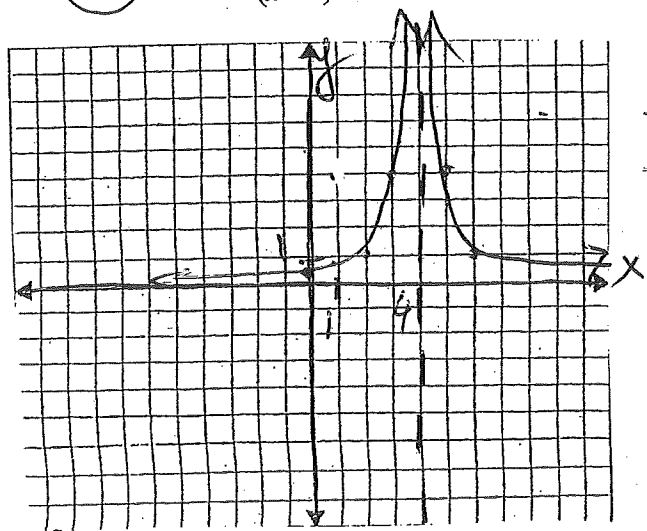
vert. as. $x=12$
hor. as. $y=1$

x	y
0	0
2	-1/5
6	-1
8	-2
10	-5
14	7
16	4
18	2



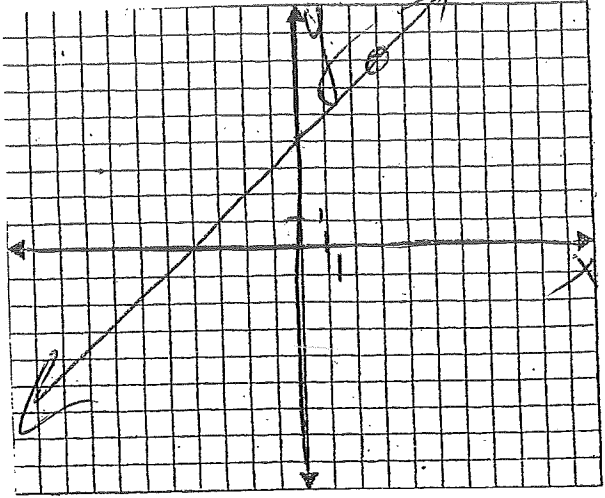
✓ (18.) $f(x) = \frac{4}{(x-4)^2}$

vert. as. $x=4$
hor. as. $y=0$



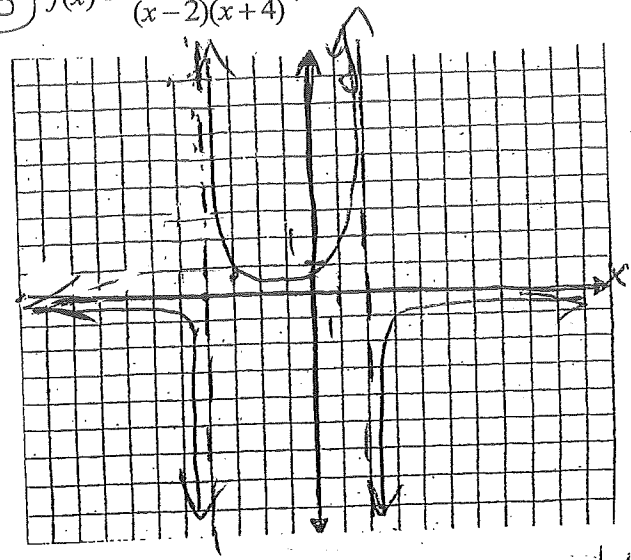
x	y
0	1/4
1	1
2	1
3	4
5	4
6	1

19. $f(x) = \frac{x^2+x-12}{x-3} = \frac{(x+4)(x-3)}{x-3}$



hole: $x=3$

20. $f(x) = \frac{-2}{(x-2)(x+4)}$

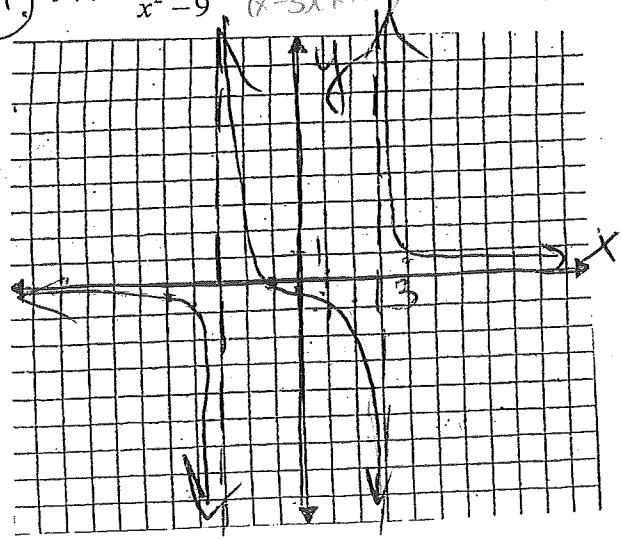


ver. as.: $x=2$
 $x=-4$

hor. as. $y=0$

x	y
-8	$-\frac{1}{20}$
-6	$-\frac{1}{8}$
-5	$-\frac{2}{7}$
-4.5	-1.6
-3	$\frac{2}{5}$
-2	$\frac{1}{5}$
-1	$\frac{2}{3}$
0	$\frac{1}{2}$
1	$\frac{2}{5}$

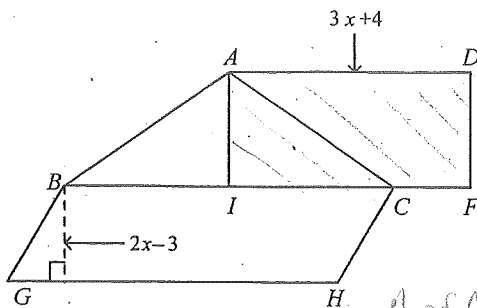
21. $f(x) = \frac{x+1}{x^2-9} = \frac{x+1}{(x-3)(x+3)}$



ver. as. $x=3$ & $x=-3$
hor. as. $y=0$

x	y
-5	$-\frac{1}{4}$
-4	$-\frac{5}{7}$
-2	$\frac{1}{5}$
-1	0
0	$-\frac{1}{9}$
1	$-\frac{1}{7}$
2	$-\frac{3}{5}$
4	$\frac{5}{7}$
5	$\frac{6}{27}$

22. Rectangle $ADFI$ has an area of $6x^2 + x - 15$ square meters and a length of $3x + 4$ meters. Parallelogram $BCHG$ has an area of $12x^2 - 11x - 36$ square meters and a height of $2x - 3$ meters. Find the area of the triangle ABC .



$$AI = \frac{6x^2 + x - 15}{3x + 4} \quad \begin{matrix} A = w \\ l \end{matrix}$$

$$BC = \frac{12x^2 - 11x - 36}{2x - 3} \quad \begin{matrix} A = l \\ b \end{matrix}$$

$$A \text{ of } \triangle ABC = \frac{1}{2}bh = \frac{1}{2}(BC)(AI)$$

$$A_{\triangle} = \frac{AI \cdot BC}{2} = \frac{1}{2} \cdot \frac{6x^2 + x - 15}{3x + 4} \cdot \frac{12x^2 - 11x - 36}{2x - 3} =$$

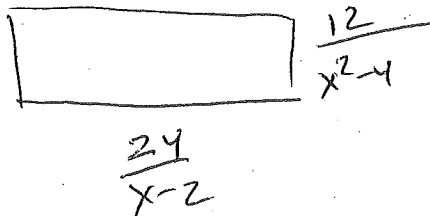
$$= \frac{1}{2} \cdot \frac{(3x+5)(2x-3)}{(3x+4)(2x-3)} \cdot \frac{(3x+4)(4x-9)}{(2x-3)} =$$

$$= \frac{(3x+5)(4x-9)}{2} \text{ m}^2$$

23. A raffle prize of $\frac{14x^2}{15}$ dollars is to be divided among $7x$ people. Write an expression for the amount of money that each person will receive.

$$\frac{14x^2}{15} \div 7x = \frac{14x^2}{15} \cdot \frac{1}{7x} = \left(\frac{2x}{15}\right)$$

24. Patrick is constructing a model of a building. The length of the windows in the building can be modeled by the expression $\frac{24}{x-2}$, and the width of the windows can be modeled by $\frac{12}{x^2-4}$. Write an expression for the perimeter of a window in Patrick's model.



$$P = 2 \cdot \left(\frac{24}{x-2} + \frac{12}{x^2-4} \right) =$$

$$= 2 \cdot \frac{24x + 48 + 12}{(x-2)(x+2)} = \frac{2 \cdot (24x + 60)}{(x-2)(x+2)} =$$

$$= \frac{2 \cdot 12(2x+5)}{(x-2)(x+2)} = \frac{24(2x+5)}{(x-2)(x+2)}$$

$$P = 2 \left(\frac{24}{x-2} + \frac{12}{(x+2)(x-2)} \right)$$

$$2 \left(\frac{24(x+2) + 12}{(x+2)(x-2)} \right)$$

$$2 \left(\frac{24x + 48 + 12}{(x+2)(x-2)} \right)$$

$$\frac{2}{1} \left(\frac{24x + 60}{(x+2)(x-2)} \right)$$

$$\frac{24(2x+5)}{(x+2)(x-2)}$$

25. Triangle B has an area of $6x^2 + x - 15$ square centimeters and a height of $3x + 4$ centimeters. Triangle C has an area of $12x^2 - 11x - 36$ square centimeters and a base of $2x - 3$ centimeters. Find the area of the rectangle A.

ΔB
 $A = \frac{1}{2}bh$
 $\frac{2A}{h} = b$

ΔC
 $A = \frac{1}{2}bh$
 $\frac{2A}{b} = h$

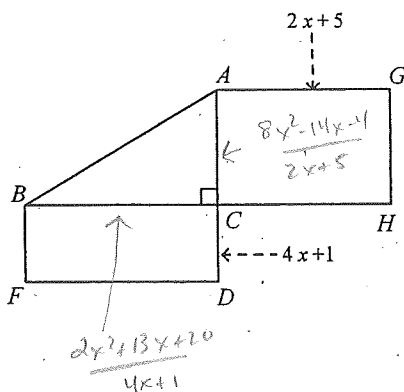
$\frac{2(6x^2 + x - 15)}{3x + 4}$

$\frac{2(12x^2 - 11x - 36)}{2x - 3}$

Area Rect A = l.w

$A = \frac{2 \cdot (3x+5)(2x-3) \cdot 2(3x+4)(4x-9)}{(3x+4)(2x-3)} = (4(3x+5)(4x-9)) \text{ cm}^2$

26. Rectangle AGHC has an area of $8x^2 - 14x - 4$ square centimeters and a length of $2x + 5$ centimeters. Rectangle BCDF has an area of $2x^2 + 13x + 20$ square centimeters and a width of $4x + 1$ centimeters. Find the area of the triangle ABC.



$A = \frac{1}{2} \cdot \frac{8x^2 - 14x - 4}{2x + 5} \cdot \frac{2x^2 + 13x + 20}{4x + 1}$

$= \frac{1}{2} \cdot \frac{(4x^2 - 7x - 2)(2x + 5)(x + 4)}{2x + 5} \cdot \frac{(4x + 1)(x - 2)(x + 4)}{4x + 1}$

$= \frac{(4x + 1)(x - 2)(x + 4)}{4x + 1}$

$= (x - 2)(x + 4) \text{ cm}^2$

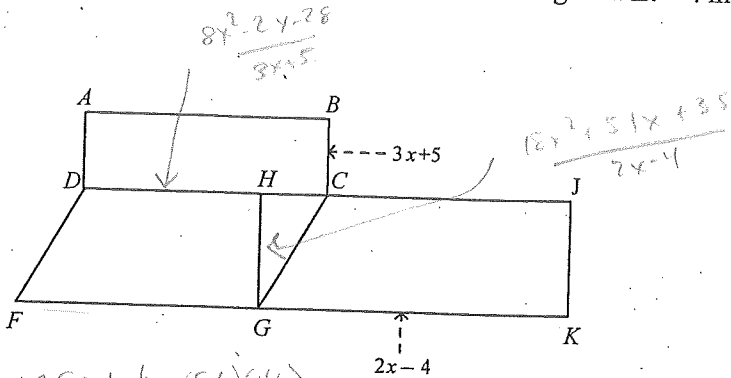
27. The length of a rectangle is $\frac{10x^2 - 13x - 3}{4x + 3}$ inches and its width is $\frac{4x^2 + 11x + 6}{2x - 3}$ inches. Find the area of the rectangle.

$A = l.w$

$\frac{10x^2 - 13x - 3}{4x + 3} \cdot \frac{4x^2 + 11x + 6}{2x - 3} = \frac{(5x + 1)(2x - 3)(4x + 3)(x + 2)}{(4x + 3)(2x - 3)}$

$= (5x + 1)(x + 2) \text{ in}^2$

28. Rectangle $ABCD$ has an area of $8x^2 - 2x - 28$ square meters and a width of $3x + 5$ meters. Rectangle $HJKG$ has an area of $18x^2 + 51x + 35$ square meters and a length of $2x - 4$ meters. Find the area of the parallelogram $DCGF$.



$$\begin{aligned}
 \square DCGF &= b \cdot h = (FG)(HG) \\
 A &= \frac{8x^2 - 2x - 28}{3x + 5} \cdot \frac{18x^2 + 51x + 35}{2x - 4} = \frac{(4x+7)(x-2)}{(3x+5)} \cdot \frac{(3x+5)(6x+7)}{(x-2)} \\
 &= \frac{(4x+7)(x-2)(6x+7)}{(x-2)} = (4x+7)(6x+7) \text{ m}^2
 \end{aligned}$$

29. Denise can clean the house in 4 hours, whereas her sister Angela can do the same work in 3 hours. They decide to work together so that they can finish the work before their aunt arrives. How long will it take them to clean the house together?

	Denise	Angela
rate	$\frac{1}{4}$	$\frac{1}{3}$
time	t	t

$$\begin{aligned}
 \frac{1}{4}t + \frac{1}{3}t &= 1 \\
 \frac{7t}{12} &= 1 & t &= \frac{12}{7} \text{ hour.}
 \end{aligned}$$

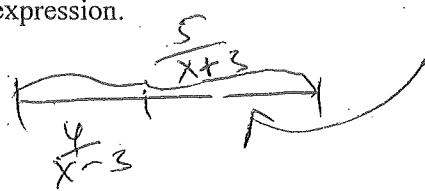
$$\frac{1}{4}t + \frac{1}{3}t = 1 \quad 1 \frac{5}{7} \text{ hour.}$$

$$\frac{3t}{12} + \frac{4t}{12} = 1$$

$$7t = 12$$

$$t = \frac{12}{7} \text{ of an hour.}$$

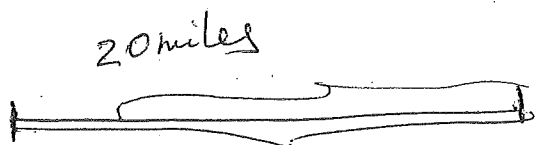
30. A board of length $\frac{5}{x+3}$ inches was cut into two pieces. If one piece is $\frac{4}{x-3}$ inches, express the length of the other board as a rational expression.



$$\frac{5}{x+3} - \frac{y}{x-3} = \frac{5(x-3) - y(x+3)}{(x+3)(x-3)}$$

$$= \frac{5x - 15 - yx - 12}{x^2 - 9} = \frac{x - 27}{x^2 - 9}$$

31. Austin rented a houseboat on a river. He traveled a distance of 30 miles downstream and 20 miles upstream. The speed of the current is 2 miles per hour. If the speed of the boat in still water is x miles per hour, write an expression for the total time taken by Austin to travel a distance of 50 miles.



30 miles

$$\text{time} = \frac{30}{x+2} + \frac{20}{x-2} =$$

(Note: $x+2$ is labeled "w/ current" and $x-2$ is labeled "against current". Arrows point from "downstream" to the 30 and from "upstream" to the 20.)

$$d = rt$$

$$t = \frac{d}{r}$$

$$= \frac{30(x-2) + 20(x+2)}{x^2 - 4} = \frac{30x - 60 + 20x + 40}{x^2 - 4} =$$

$$= \frac{50x - 20}{x^2 - 4}$$

8-4 Practice**Direct, Joint, and Inverse Variation**

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$ joint

2. $p = 4s$ direct
(4)

3. $L = \frac{5}{k}$ inverse
(5)

4. $xy = 4.5$ inverse
(4.5)

5. $\frac{C}{d} = \pi$
 $C = \pi d$ direct
(π)

6. $2d = mn$
 $d = \frac{1}{2}mn$ joint
($\frac{1}{2}$)

7. $\frac{1.25}{g} = h$
 $1.25 = hg$ inverse
(1.25)

8. $y = \frac{3}{4x}$ inverse
($\frac{3}{4}$)

Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$. (24)

10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$. (1.5)

11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$. (396)

12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$. ($\frac{28}{3}$)

13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$. (288)

14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$. (240)

15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$. (2)

16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$. ($\frac{64}{3}$)

17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$. (6)

18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$. (-21.6)

19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$. (3)

20. **GASES** The volume V of a gas varies inversely as its pressure P . If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury.
 $xy = k$
 $\sqrt{P} = k$ (500 cm³)

21. **SPRINGS** The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?
 $y = kx$ $S = kF$ (12 in)

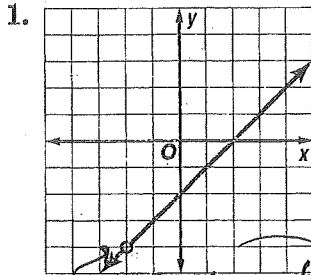
22. **GEOMETRY** The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters?
 $y = kx^2$ $A = khB$ (100 m²)

8-5 Practice

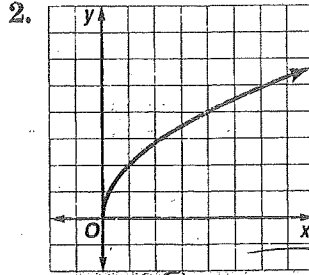
Classes of Functions

Key

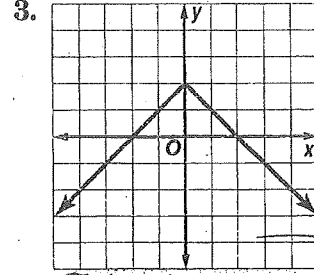
Identify the type of function represented by each graph.



rational



square root



absolute value

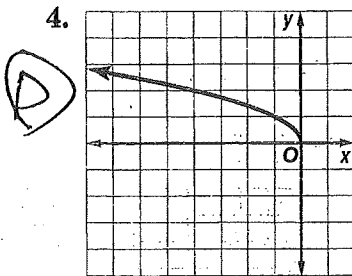
Match each graph with an equation below.

A. $y = |2x + 1|$

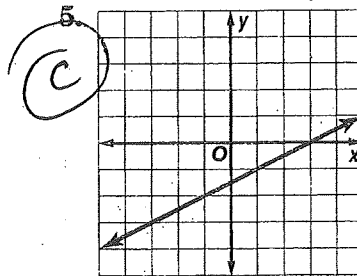
B. $y = [2x + 1]$

C. $y = \frac{x-3}{2}$
 $y = \frac{1}{2}x - \frac{3}{2}$

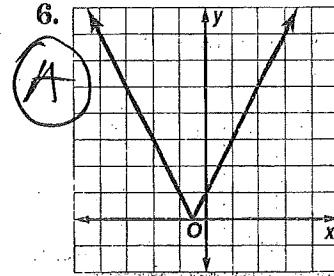
D. $y = \sqrt{-x}$



D



C

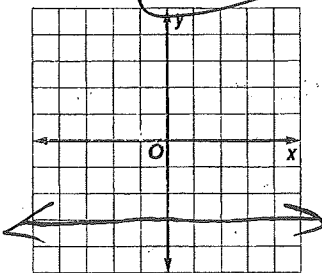


A

Identify the type of function represented by each equation. Then graph the equation.

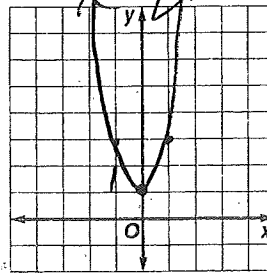
7. $y = -3$

const



8. $y = 2x^2 + 1$

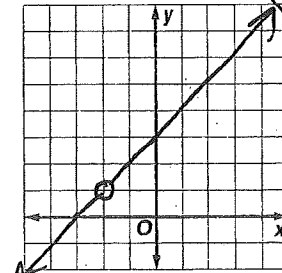
quadratic



9. $y = \frac{x^2 + 5x + 6}{x + 2}$

rational

$x \neq -2$
 $\frac{(x+2)(x+3)}{x+2}$



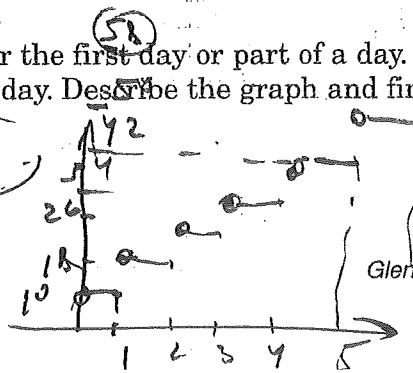
10. **BUSINESS** A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function.

It is a parabola

11. **PARKING** A parking lot charges \$10 to park for the first day or part of a day. After that, it charges an additional \$8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days. = 7 days

Step; \$58

Day 1 = \$10
Day 2-7 = (6)(8) = 48

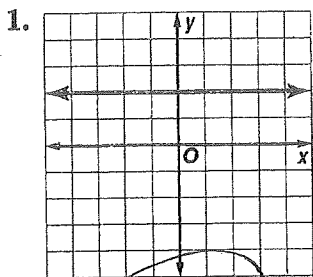


8-5 Skills Practice

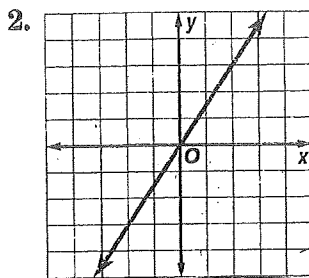
Classes of Functions

Key

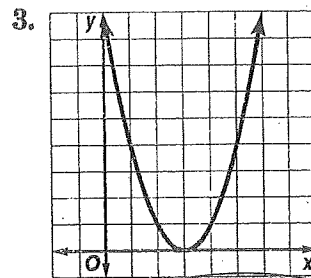
Identify the type of function represented by each graph.



constant



direct var.



quadratic

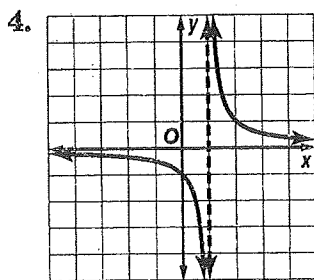
Match each graph with an equation below.

A. $y = |x - 1|$

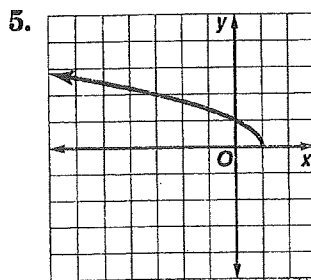
B. $y = \frac{1}{x - 1}$

C. $y = \sqrt{1 - x}$

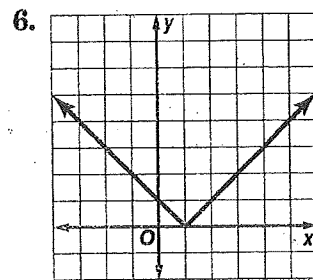
D. $y = [|x|] - 1$



B



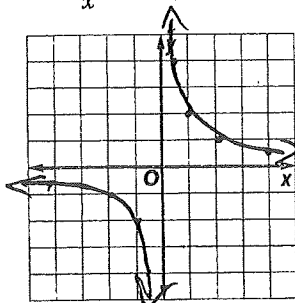
C



A

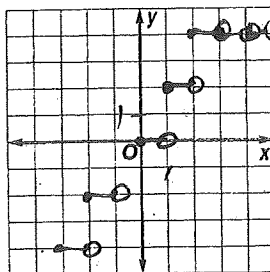
Identify the type of function represented by each equation. Then graph the equation.

7. $y = \frac{2}{x}$



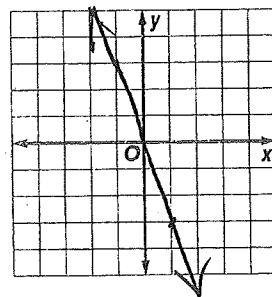
inverse variation
or
rational.

8. $y = 2[x]$



greatest
integer

9. $y = -3x$



direct
var.

x	y
1/2	4
1	2
2	1
4	1/2
-1	-2
-2	-1

Describe how the graphs of $f(x)$ + $g(x)$ are related.

13) $f(x) = x$ $g(x) = x + 6$

translated 6 units up

14) $f(x) = x^2$ $g(x) = \frac{3}{4}x^2$

compressed vertically by $\frac{3}{4}$

15) $f(x) = |x|$ $g(x) = |5x|$

compressed (shrunk) horizontally by $\frac{1}{5}$

16) $f(x) = x^3$ $g(x) = (x-5)^3$

translated 5 unit right

17) $f(x) = \frac{1}{x}$ $g(x) = \frac{3}{x}$ $3(\frac{1}{x})$

stretched vertically by 3

18) $f(x) = [x] + 1$ $g(x) = -[x] - 1$

reflected over x-axis

19) $f(x) = \sqrt{x}$ and $g(x) = \sqrt{.4x} + 3$

neg \rightarrow reflected over x-axis

$.4 \rightarrow$ stretched horizontally by $\frac{5}{2}$

$+3 \rightarrow$ translated up 3

20) $f(x) = x^2$

a) $y = -(1.5x)^2$

reflected over x-axis
horizontal stretch by $\frac{2}{3}$

b) $y = 4(x-3)^2$

vertical stretch by 4
translated 3 right

c) $y = \frac{1}{2}x^2 - 5$

vertically stretch by $\frac{1}{2}$
translated 5 down

21) $f(x) = |x|$

a) $y = |0.2x|$

horizontal stretch by 5

b) $y = 7|x| - 0.4$

vertical stretch by 7
translation 0.4 down

c) $y = -9|x+1|$

reflection over x-axis
vertical stretch by 9
translation 1 left

22) $f(x) = x^3$

a) $y = (x+2)^3 - 5$

translated 2 left + 5 down

b) $y = -(0.8x)^3$

reflected over x-axis
horizontal stretch by $5/4$

c) $y = (\frac{5}{3}x)^3 + 2$

horiz. stretch by $3/5$
translated 2 up

23) $f(x) = \sqrt{x}$

a) $y = \frac{1}{3}\sqrt{x+2}$

vertical stretch by $\frac{1}{3}$
translated 2 left

b) $y = \sqrt{-x} - 7$

reflected over y-axis
translated 7 down

c) $y = 4 + 2\sqrt{x-3}$

vertical stretch by 2

$\hookrightarrow y = 2\sqrt{x-3} + 4$

translated 3 right + 4 up

24) $f(x) = \frac{1}{x}$

a) $y = \frac{1}{0.5x}$

horizontal stretch by 2

b) $y = \frac{1}{6x} + 8$

horiz. stretch by $\frac{1}{6}$
translated 8 up

c) $y = \frac{1}{|x|}$

left of y-axis removed
remaining graph reflected over y-axis

25) $f(x) = \lfloor x \rfloor$

a) $y = \lfloor \frac{5}{2}x \rfloor - 3$

horiz. stretch by $2/5$
translated 3 down

b) $y = -0.75 \lfloor x \rfloor$

vert. stretch by 0.75
reflected over x-axis

c) $y = \lfloor |x| - 4 \rfloor$

portion of graph left of y-axis removed
remaining portion reflected over y-axis
then,
translated 4 right

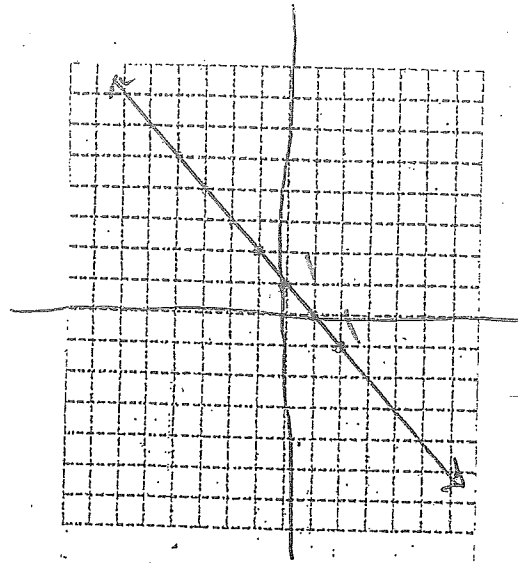
8.5 d = cont.

28) $f(x) = -(x+4) + 5$

linear

$$-x - 4 + 5$$

$$-x + 1$$

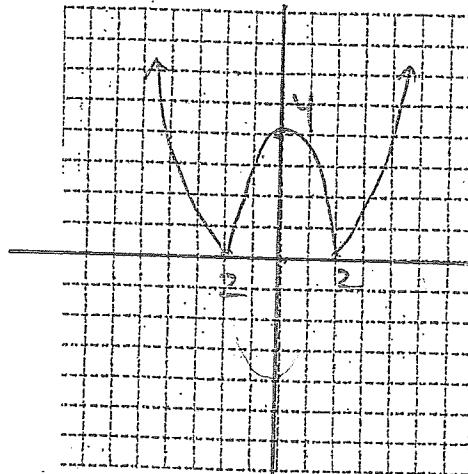


29) $f(x) = |x^2 - 4|$ $y = x^2$

$y = x^2$
 trans. 4 down
 then reflected over x-axis

$$y = x^2 - 4$$

$$y = x^2 - 4$$



30) $f(x) = (0.5x - 1)^3$

$y = x^3$: $f(x) = (\frac{1}{2}(x-2))^3$
 horiz stretch by 2

trans 2 right

$$y = x^3$$

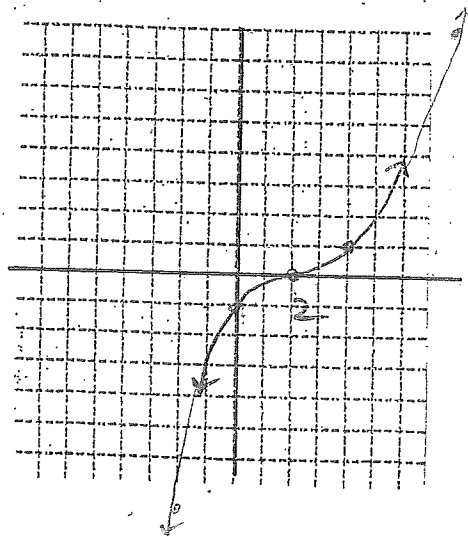
-2	-8
-1	-1
0	0
1	1
2	8

$$x/2$$

-4	-8
-2	-1
0	0
2	1
4	8

$$x+2$$

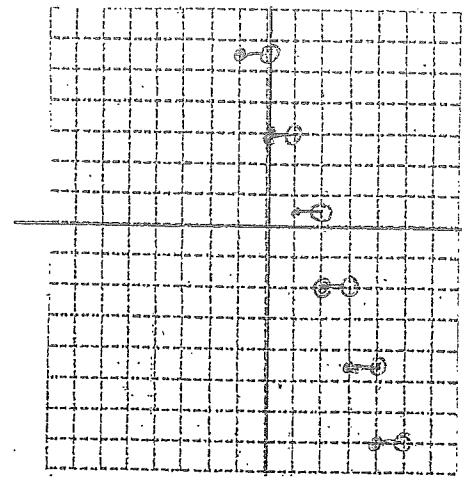
-2	-8
0	-1
2	0
4	1
8	8



31) $f(x) = -2.5 [x] + 3$

step $y = [x]$

reflected over y-axis
 vert. stretch by 2.5
 translated 3 up



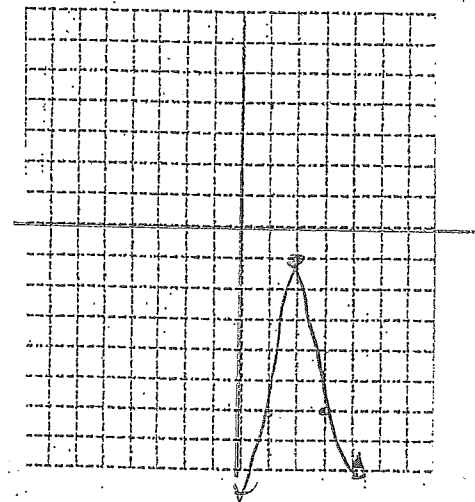
32) $f(x) = -4|x-2| - 1$

$y = |x|$

reflected over x-axis
 vert stretch by 4
 trans. 2 right, 1 down

-2	2
-1	1
0	0
1	1
2	2

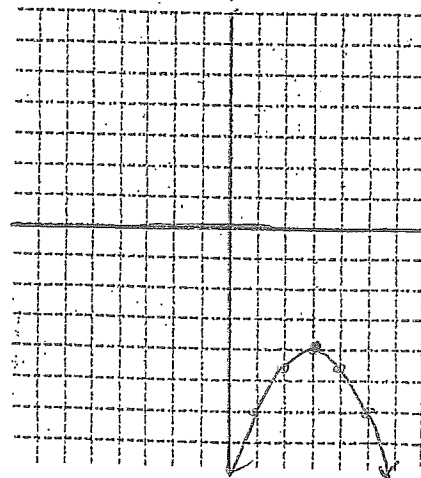
$y(-4)$	$x+2$	$y-1$
-2, -8	0	-9
-1, -4	1	-5
0, 0	2	-1
1, 4	3	3
2, 8	4	7



33) $f(x) = -\frac{1}{2}(x-3)^2 - 4$

reflected over x-axis
 vert stretch by $\frac{1}{2}$
 translated 3 right + 4 down

$y = x^2$	$y(-\frac{1}{2})$	$x+3$	$y-4$
-2, 4	-2, -2	+1	-6
-1, 1	-1, -1/2	2	-4 1/2
0, 0	0, 0	3	-4
1, 1	1, -1/2	4	-4 1/2
2, 4	2, -2	5	-6



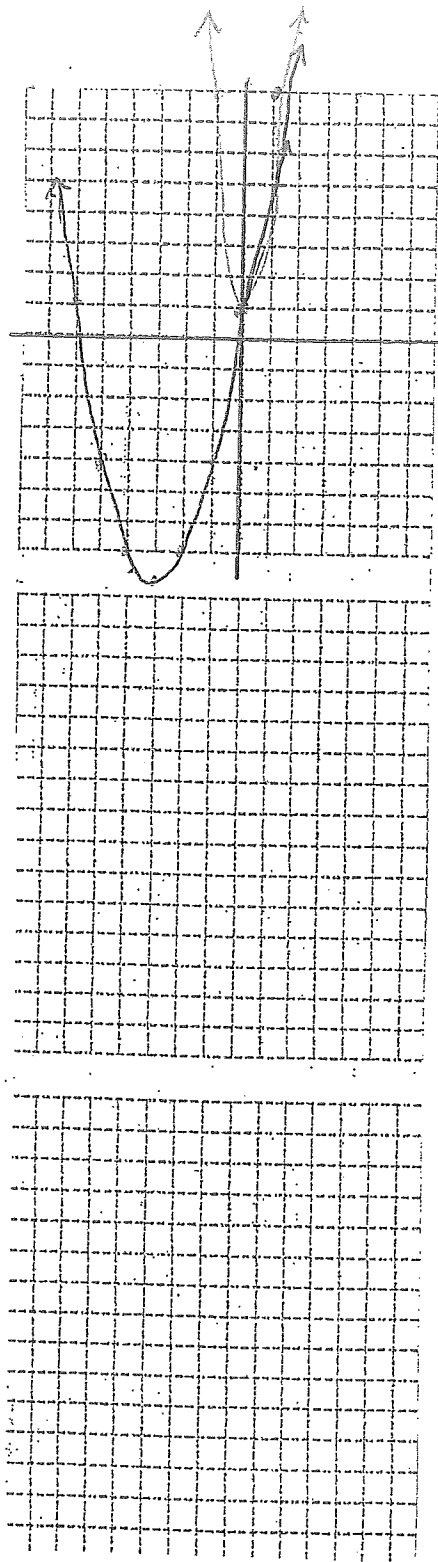
34) graph $y = f(x)$
↓ $y = f(|x|)$

$$f(x) = (x+3)^2 - 8$$

$$(|x|+3)^2 - 8$$

$y = x^2$ 3 left, 8 down

-2, 4	-5, -4
-1, 1	-4, -7
0, 0	-3, -8
1, 1	-2, -7
2, 4	-1, -4
	0, 1
	1, 9
	2, 16



8-6**Skills Practice****Solving Rational Equations and Inequalities****Key**

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$

(-1)

2. $2 = \frac{4}{n} + \frac{1}{3}$

 $\frac{12}{5}$

3. $\frac{9}{3x} = \frac{-6}{2}$

(-1)

4. $3 - z = \frac{2}{z}$

{1, 2}

5. $\frac{2}{d+1} = \frac{1}{d-2}$

(5)

6. $\frac{s-3}{5} = \frac{8}{s}$

{-5, 8}

7. $\frac{2x+3}{x+1} = \frac{3}{2}$

(-3)

8. $-\frac{12}{y} = y - 7$

{3, 4}

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$

(8)

10. $\frac{3}{k} - \frac{4}{3k} > 0$

(0, ∞)

11. $2 - \frac{3}{v} < \frac{5}{v}$

(0, 4)

12. $n + \frac{3}{n} < \frac{12}{n}$

 $(-\infty, -3) \cup (0, 3)$

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$

(0, 1)

14. $\frac{1}{2x} < \frac{2}{x} - 1$

(0, $\frac{2}{3}$)

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$

(3)

16. $\frac{3b-2}{b+1} = \frac{4}{1} - \frac{b+2}{b-1}$

(4)

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$

(-5)

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$

 $\frac{2}{5}$

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$

(-4)

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

~~0~~

21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

~~0~~

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$

(2)

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$

(-6)

24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$

(5)

8-6 Practice

Key

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ (16)

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ (-1, 2)

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ (-2/3, 4)

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ (4)

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ $(-\infty, 5) \cup (5, \infty)$

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ (5/2)

7. $\frac{5}{t} < \frac{9}{2t+1}$ $(-\infty, -5) \cup (-2, 0)$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ (11/5)

9. $\frac{4}{w-2} = \frac{-1}{w+3}$ -2

10. $5 - \frac{3}{a} < \frac{7}{a}$ (0, 2)

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ (0, 7)

12. $8 + \frac{3}{y} > \frac{19}{y}$ $(-\infty, 0) \cup (2, \infty)$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ $(-\infty, 0) \cup (3, \infty)$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ \emptyset

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ (-1)

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ (-2)

17. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ (3/2)

18. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ (5/3, 5/9)

19. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ (7)

20. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ (-1, -2)

21. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ (0)

22. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ \emptyset

23. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ $r \neq \pm 4$

24. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ (-2)

$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? Is this a reasonable answer? Explain.

$$\frac{9+x}{19+x} = \frac{6}{10}$$

$$x=6$$

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? Is this a reasonable answer? Explain.

$$\frac{1}{p} + \frac{1}{5} = \frac{1}{4}$$

$$\frac{1}{p} = \frac{1}{20}$$

$$p = 20 \text{ cm}$$

Review (8.4-8.6)

Key

1. The average American drinks about eight servings of hydrated beverages everyday.

A.) Write an equation to represent the total servings of hydrated beverages consumed w by m household members during a period of d days.

$$W = 8md$$

B.) Each member of a household of four members drinks the same amount of hydrated beverages each day as the average American. How many servings of hydrated beverages (w) would the members of the household consume in a week?

$$m = 4$$

$$d = 7$$

$$W = 4 \cdot 8 \cdot 7 = 224$$

2. Monica runs on a treadmill at an average speed of 7.5 miles per hour for 15 minutes. If, the next day, she runs the same distance at a speed of 8 miles per hour, what is the average time taken?

avg. dist.
 $(7.5 \frac{m}{hr})(.25 hr)$
 $= 1.875 \text{ miles}$

$$d = 7.5 \cdot \frac{1}{4} = 7 \frac{1}{2} \cdot \frac{1}{4} = \frac{15}{8}$$

$$\frac{d}{8} = \frac{15}{8} \div 8 = \frac{15}{64} \text{ hour} = 0.23 \text{ hour}$$

$$\frac{15}{60} = \frac{1}{4} \text{ hour}$$

3. Many areas of Northern California depend on the snowpack of the Sierra Nevada Mountains for their water supply. If 300 cubic centimeters of snow will melt to 33 cubic centimeters of water, how much water does 600 cubic centimeters of snow produce?

1.875 miles
 $8 \frac{m}{hr}$
 $= .23 \text{ hr}$
 $= 14 \text{ min}$

$$\frac{300}{33} = \frac{600}{x}$$

$$\frac{1}{33} = \frac{2}{x}$$

$$x = 66 \text{ cm}^3$$

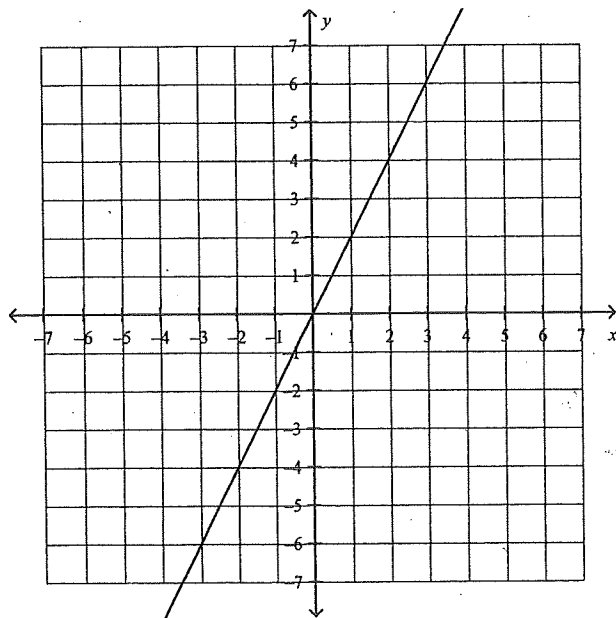
4. Megan drove from her house to work at an average speed of 45 miles per hour. The drive took her 20 minutes. If the drive home took her 30 minutes and she used the same route in reverse, what was her average speed going home?

$$d = 45 \cdot \frac{1}{3} \text{ hr} = 15 \text{ miles}$$

$$\text{speed} = \frac{15 \text{ miles}}{\frac{1}{2} \text{ hr}} = 30 \frac{\text{miles}}{\text{hr}}$$

Identify the type of function represented by the graph.

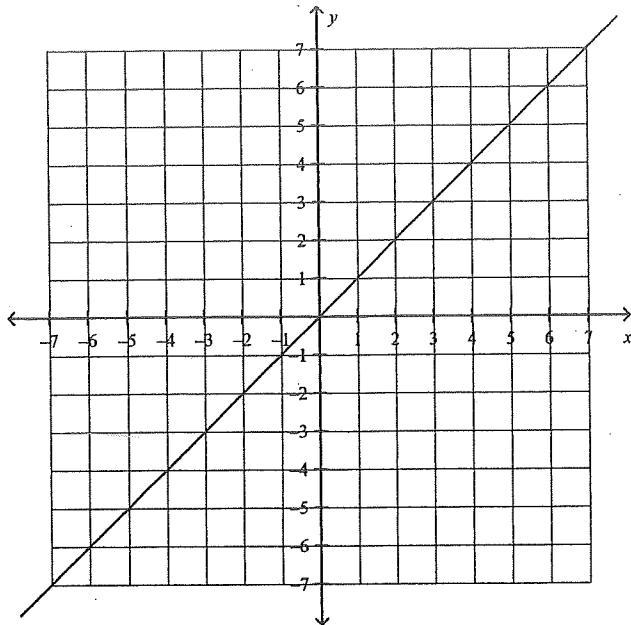
5



D

- a. absolute value function
- b. constant function
- c. square root function
- d. direct variation function

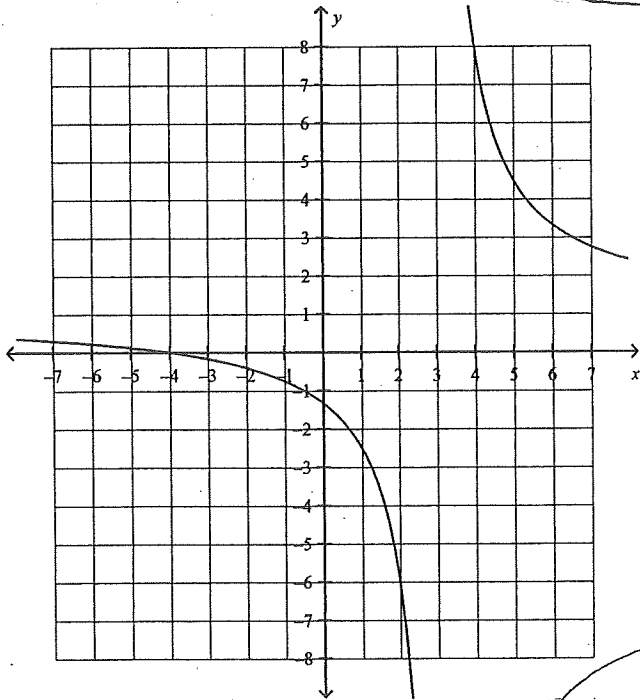
36.



- a. square root function
- b. constant function
- c. inverse variation function
- d. identity function

D

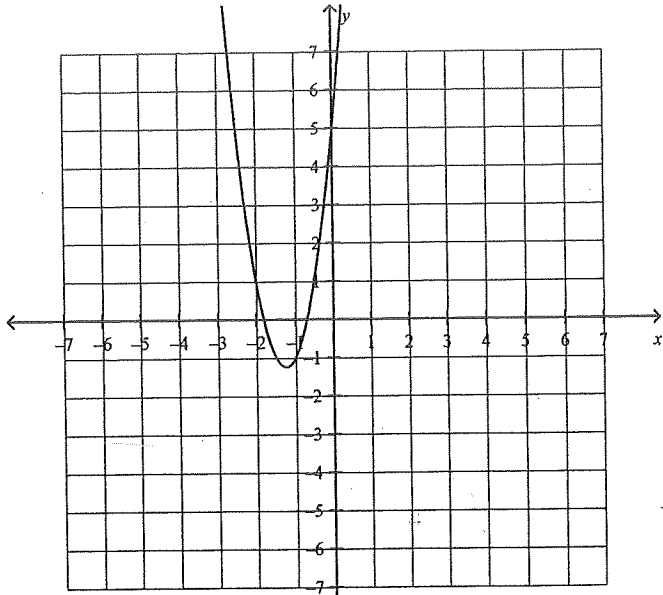
37.



- a. quadratic function
- b. square root function
- c. inverse variation function
- d. rational function

AD

6

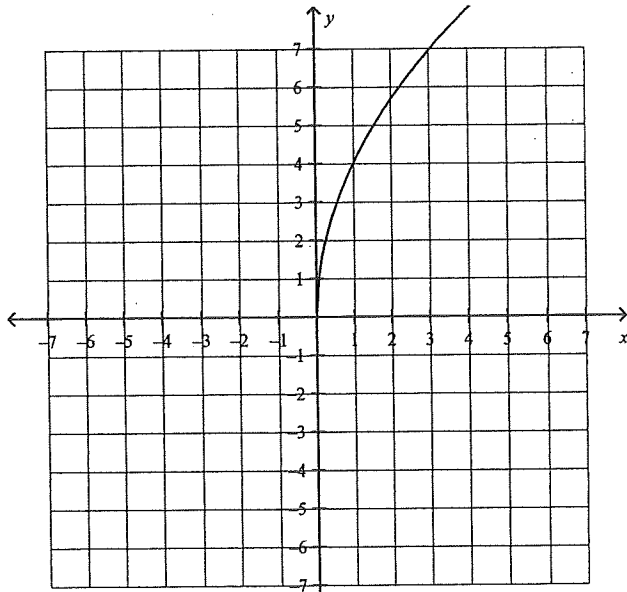


C

- a. absolute value function
- b. inverse variation function

- c. quadratic function
- d. rational function

7

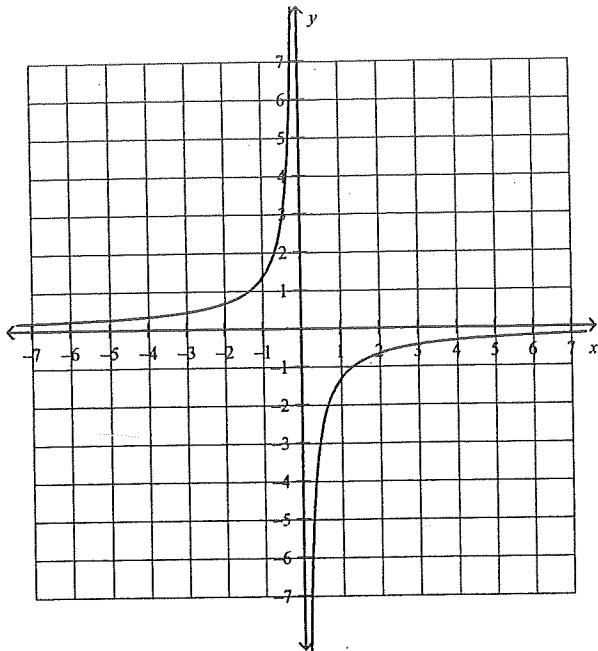


A

- a. square root function
- b. absolute value function

- c. direct variation function
- d. rational function

8.

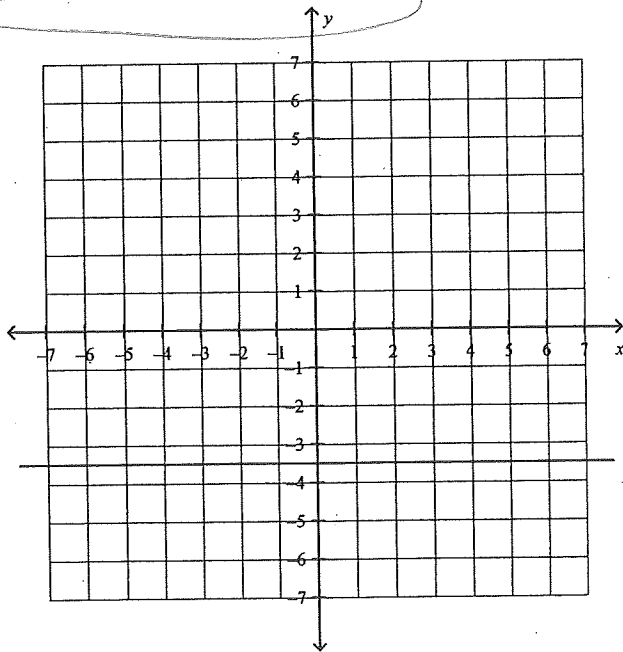


B

- a. absolute value function
- b. inverse variation function

- c. direct variation function
- d. constant function

9.

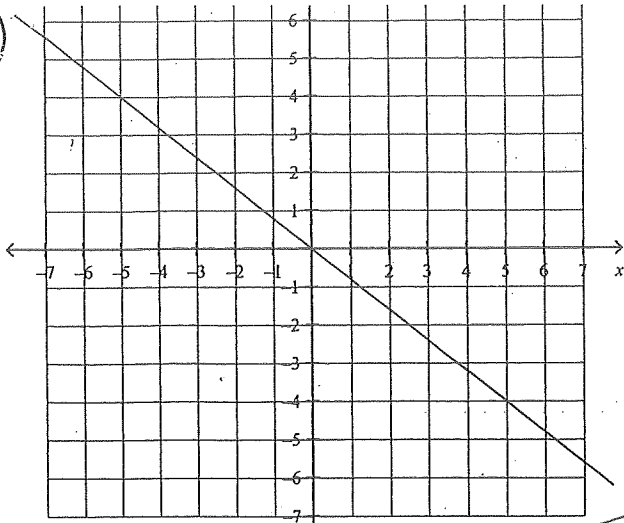


d

- a. absolute value function
- b. identity function

- c. direct variation function
- d. constant function

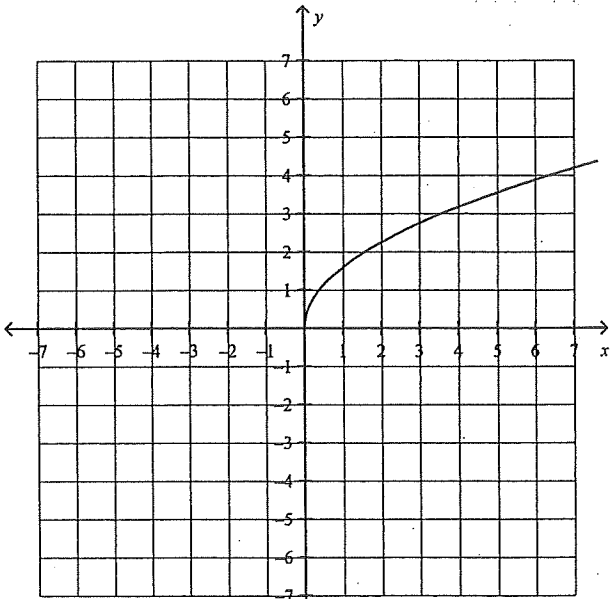
10.



- a. square root function
- b. rational function
- c. direct variation function
- d. inverse variation function

C

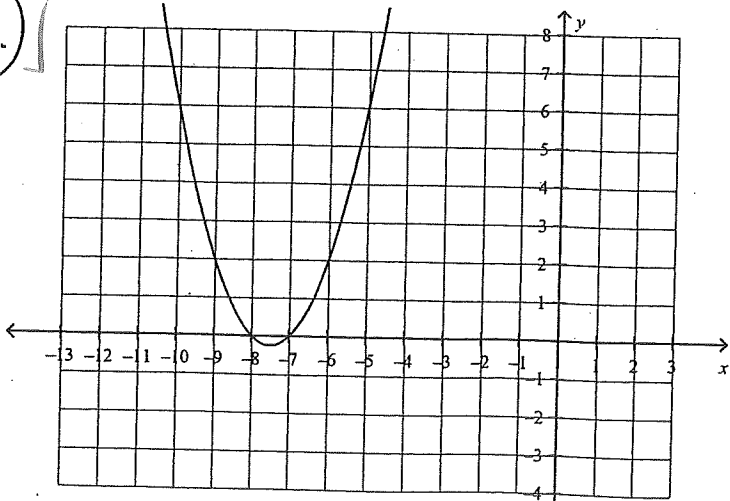
11.



- a. square root function
- b. quadratic function
- c. constant function
- d. identity function

A

12.



- a. absolute value function
- b. quadratic function
- c. rational function
- d. inverse variation function

B

Identify the type of function represented by the equation.

13) $y = \frac{x+4}{x-2}$

(D)

- a. direct variation function
b. constant function

- c. quadratic function
d. rational function

14) $y = \sqrt{36x}$

(A)

- a. square root function
b. rational function

- c. inverse variation function
d. quadratic function

15) $y = -\frac{6}{x}$

(C)

- a. absolute value function
b. direct variation function

- c. inverse variation function
d. identity function

16) $y = |8x|$

(A)

- a. absolute value function
b. identity function

- c. constant function
d. direct variation function

17) $y = -10$

(C)

- a. absolute value function
b. inverse variation function

- c. constant function
d. direct variation function

18) $y = x^2 + 5x + 6$

(A)

- a. quadratic function
b. identity function

- c. rational function
d. absolute value function

19) $y = -7.5x$

(B)

- a. absolute value function
b. direct variation function

- c. identity function
d. constant function

20) If y varies directly as x and $y = 30$ when $x = -10$, find y when $x = 56$.

$y = kx$
 $30 = k(-10)$
 $k = -3$
 $y = -3 \cdot 56 = -168$

21) If y varies directly as x and $y = 28$ when $x = 4$, find y when $x = 20$.

$28 = k \cdot 4$, $k = 7$
 $y = 7 \cdot 20$, $y = 140$

22) Suppose y varies jointly as x and z . Find y when $x = 2$ and $z = 11$, if $y = 160$ when $x = 3$ and $z = 8$. Round your answer to the nearest hundredth, if necessary.

$y = kxz$
 $160 = k \cdot 3 \cdot 8$
 $k = \frac{20}{3}$
 $y = \frac{20}{3} \cdot 2 \cdot 11 = \frac{440}{3} \approx 146.67$

23) Suppose y varies jointly as x and z . Find y when $x = -4$ and $z = 25$, if $y = 206$ when $x = -6$ and $z = -11$. Round your answer to the nearest hundredth, if necessary.

$y = k \cdot x \cdot z$
 $206 = k \cdot (-6) \cdot (-11)$
 $k = \frac{206}{66} = \frac{103}{33}$
 $y = \frac{103}{33} \cdot (-4) \cdot 25 = -\frac{10300}{33} \approx -312.12$

24) If y varies inversely as x and $y = 194$ when $x = -13$, find y when $x = 50$. Round your answer to the nearest hundredth, if necessary.

$y = \frac{k}{x}$
 $194 = \frac{k}{-13}$
 $k = -194 \cdot 13$
 $y = \frac{-194 \cdot 13}{50} = -50.44$

Solve the given equation. Round answers to the nearest hundredth, if necessary.

✓ (25) $\frac{x}{x+2} = \frac{2}{19}$

$x \neq -2$

$19x = 2x + 4$
 $17x = 4$

$\left\{ \frac{4}{17} \right\} =$

✓ (26) $27 = \frac{4}{s} + 14$

$\frac{13}{1} = \frac{4}{s}$, $s = \frac{4}{13} = 3.25$

$= 0.24$

Solve the inequality. Check your solution.

✓ (27) $\frac{5}{b+1} > 5$, $\frac{5}{b+1} - 5 > 0$, $\frac{5 - 5b - 5}{b+1} > 0$

$-\frac{5 \cdot b}{b+1} > 0$, $-\frac{5}{2} < 0$

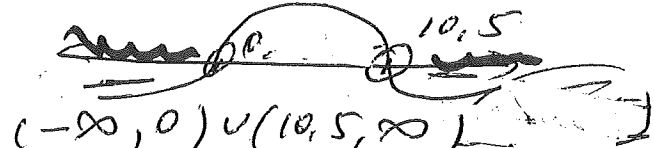
✓ (28) $\frac{8}{2p} + \frac{5}{4p} < \frac{1}{2}$



$(-\infty, 0)$

$\frac{16+5}{4p} < \frac{2p}{4p}$

$\frac{21-2p}{4p} < 0$



✓ (29) Carl drove from his house to work at an average speed of 35 miles per hour. The drive took him 25 minutes. If the drive home took him 30 minutes and he used the same route in reverse, what was his average speed going home?

$d = 35 \cdot \frac{25}{60} = 35 \cdot \frac{5}{12} = \frac{35 \cdot 5}{12}$, $speed = \frac{35 \cdot 5}{12} \cdot \frac{6}{6} = \frac{35 \cdot 5}{6} = 29 \frac{1}{6}$

✓ (30) In order to sustain itself in its cold habitat, a Siberian tiger requires 25 pounds of meat per day.

A.) Write an equation to represent the amount of meat needed m to sustain x Siberian tigers for d days.

$m = 25 \cdot x \cdot d$

B.) How much meat would seven Siberian tigers need for the month of April?

$x = 7$
 $d = 30$

$m = 25 \cdot 7 \cdot 30 = 5250 \text{ pounds}$

✓ (31) It has been found that the average number of daily phone calls C between two cities is directly proportional to the product of the populations P_1 and P_2 of the two cities and inversely proportional to the square of the distance d between the cities. That is, $C = \frac{kP_1P_2}{d^2}$.

A.) The distance between Albany, New York, and Cleveland, Ohio, is about 480 miles. If the average number of daily phone calls between the cities is 250,000, find the value of k and write the equation of variation. Round to the nearest thousandth. The population of Albany and Cleveland is 95,000 and 2,900,000 respectively.

$250,000 = \frac{k \cdot 95,000 \cdot 2,900,000}{480^2}$, $25 = \frac{k \cdot 950 \cdot 290}{48^2}$, $k \approx 0.209$

B.) Can you use this formula to find the average number of phone calls between two adjoining cities?

$m \quad d = 0$

32.

An aircraft flying at a steady rate travels 3840 miles with the wind. It can travel only 3260 miles against the wind in the same amount of time. If the speed of the wind is 25 miles per hour, find the speed of the aircraft in calm air.

$x + 25$ wind

$x - 25$ wind

$$\frac{3840}{x+25} = \frac{3260}{x-25}$$

$$192(x-25) = 163(x+25)$$

$$192x - 163x = 4080 + 4075$$

$$29x = 8155$$

$$x \approx 306 \frac{\text{miles}}{\text{hour}}$$

33.

Blake and Ned work for a home remodeling business. They are putting the final touches on a home they renovated. Working alone, Blake can paint one room in 9 hours. Ned can paint the same room in 6 hours. How long will it take them to paint the room if they work together?

$$\frac{t}{9} + \frac{t}{6} = 1$$

$$\text{or } \left[\frac{t}{9} + \frac{t}{6} = 1 \right] \cdot \frac{18}{1}$$

$$\frac{5t}{18} = 1$$

$$2t + 3t = 18$$

 $5t = 18$
 $t = 18/5$

$$t = \frac{18}{5} = 3 \frac{3}{5} = 3 \frac{36}{60}$$

3 hours & 36 min

34.

Sophia and Angie are distance runners. Sophia's average speed is 5.4 miles per hour and Angie averages 4.9 miles per hour. They participate in a racing competition. If Sophia finishes the race 0.6 hours ahead of Angie, find the distance they ran.



Angie Sophie

$$\textcircled{1} \frac{d}{4.9} = \frac{d}{5.4} + 0.6$$

$$\textcircled{2} \frac{d}{4.9} = \frac{d}{5.4} + \frac{3}{5}$$

$$4.9 = 4 \frac{9}{10} = \frac{49}{10}, \quad 5 \frac{4}{10} = 5 \frac{2}{5}$$

$$\textcircled{3} \frac{10d}{49} = \frac{5d}{27} + \frac{3}{5}$$

$$\textcircled{5} \frac{25d}{49 \cdot 27} = \frac{3}{5}$$

$$\textcircled{4} \frac{27 \cdot 10d - 5 \cdot 49d}{49 \cdot 27} = \frac{3}{5}$$

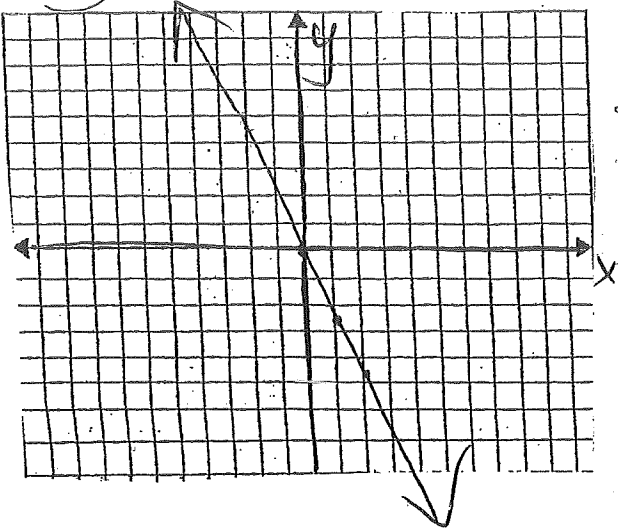
$$d = \frac{49 \cdot 27 \cdot 3}{25 \cdot 5} = \frac{3969}{125} = 31.8 \text{ miles}$$

Identify the type of function represented by the equation. Then graph the equation.

35.

$$y = -\frac{5}{2}x$$

direct
variable



36.

$$y = x^2 + 4x - 1$$

Quadratic

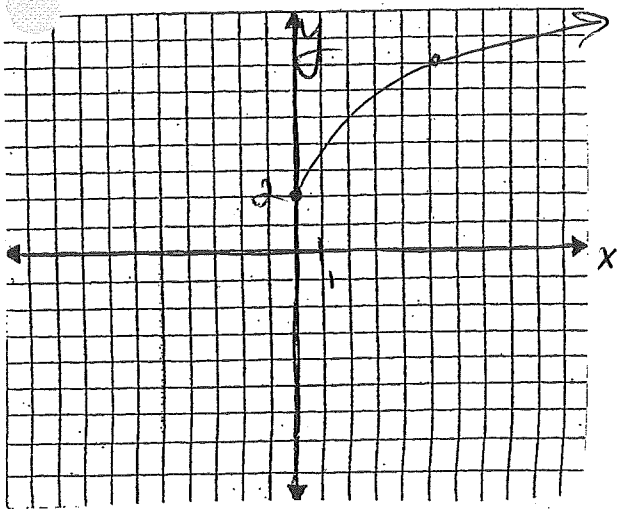


$$y = (x+2)^2 - 5 \quad \therefore \quad v(-2, -5)$$

37.

$$y = \sqrt{5x+2}$$

v. (0, 2)



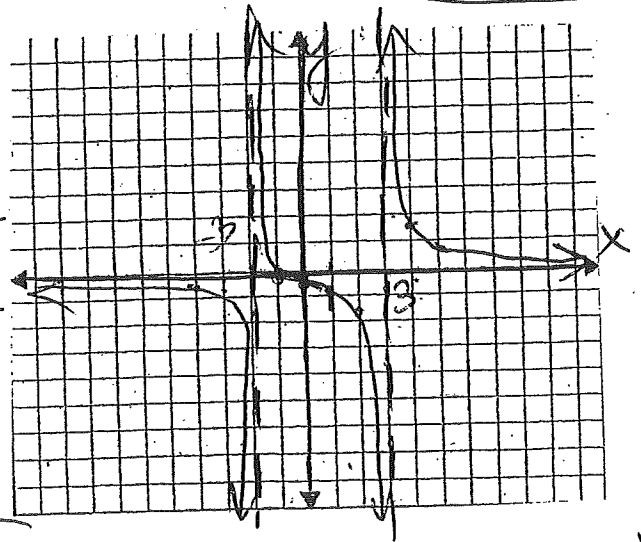
square root

x	y
0	2
5	7

38.

$$y = \frac{2(x+1)}{x^2-x-6}$$

rational
function



x	y
-4	-3/7
-3	-2/3
-1	0
0	-1/3
1	-2/3

$$2 \quad -\frac{2}{2}$$

ver. as.: $x = 3$

$x = -2$

$$y = \frac{2(x+1)}{(x-3)(x+2)}$$

$$-1.5 \quad 0.4$$

hor. as.: $y = 0$

$$4 \quad 5/3$$

$$5 \quad 6/7$$

39.

High school students are planning a field trip to a museum. The cost per student varies inversely with the number of students going on the trip. It costs \$24 per student if 60 students go on the trip. Find how much each student would pay if 45 students are going on the trip.

$$C = \frac{k}{x}$$

$$\frac{24}{1} = \frac{k}{60} \rightarrow k = 24 \cdot 60$$

$$C = \frac{1440}{45} = \$32$$

$$C = \frac{24 \cdot 60}{45} = \$32$$

$C = \frac{1440}{45} = \$32$

40.

According to Charles's Law, the volume occupied by a gas at constant pressure is directly proportional to its absolute temperature. A test conducted with a rubber ball demonstrates that the volume of the ball is 1.25 liters at 240 K. What will the volume of the rubber ball be when its temperature is 324 K?

$$V = kT$$

$$1.25 = k \cdot 240$$

$$k = \frac{1.25}{240}$$

$$V = \frac{1.25}{240} \cdot 324 = \frac{1.25 \cdot 27}{20} = 1.6875 \text{ liters}$$

41.

The pressure exerted on the walls of a container by a gas enclosed within it is directly proportional to the temperature of the gas. If the pressure is 6 pounds per square inch when the temperature is 440°F, find the pressure exerted when the temperature of the gas is 380°F.

$$P = k \cdot T$$

$$6 = k \cdot 440$$

$$k = \frac{6}{440} = \frac{3}{220}$$

$$P = \frac{3}{220} \cdot 380 = \frac{3 \cdot 19}{11} = \frac{57}{11} = 5 \frac{12}{11} \text{ lb/in}^2$$

42.

The distance a spring stretches varies directly with the weight on the spring. If a spring stretches 7.5 inches with 55 pounds attached, how far will it stretch when 38 pounds are attached?

$$d = kw$$

$$7.5 = k \cdot 55$$

$$k = \frac{7.5}{55}$$

$$d = \frac{7.5}{55} \cdot 38 = \frac{1.5 \cdot 38}{11} = \frac{57}{11} = 5.18 \text{ in.}$$

Review: Chapter 8

Key

Simplify each expression.

$$1. \frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2} = \frac{-4bc}{33a}$$

$$2. \frac{a^2 - b^2}{6b} \div \frac{a+b}{36b^2} = 6b(a-b)$$

$\frac{(a-b)(a+b)}{\cancel{6b}} \cdot \frac{3\cancel{6}b^2}{(a+b)}$

$$3. \frac{\frac{x^2 + 7x + 10}{x+2}}{\frac{x^2 + 2x - 15}{x+2}} = \frac{x+2}{x-3}$$

$$\frac{\cancel{(x+5)}(\cancel{x+2})(x+2)}{\cancel{(x+2)}(\cancel{x+5})(x-3)}$$

$$4. \frac{\frac{1}{n^2 - 6n + 9}}{n+3} = \frac{2}{n-3}$$

$$\frac{2(n^2 - 9)}{(n-3)^2(n+3)} = \frac{2(\cancel{n-3})(n+3)}{(n-3)^2(\cancel{n+3})}$$

$$5. \frac{y^2 - y - 12}{y+2} \div \frac{y-4}{y^2 - 4y - 12} = (y+3)(y-6)$$

$$(y+3)(y-6)$$

$$6. \frac{x^2 + 3x - 10}{x^2 + 8x + 15} \cdot \frac{x^2 + 5x + 6}{x^2 + 4x + 4} = \frac{x-2}{x+2}$$

7. **GEOMETRY** A triangle has an area of $2x^2 + 4x - 16$ square meters. If the base is $x-2$ meters, find the height.

$$A = \frac{1}{2}bh$$

$$\frac{2A}{b} = h$$

$$\frac{(x-2)h}{2} = 2x^2 + 4x - 16$$

$$h = \frac{2(x^2 + 2x - 8)}{x-2} = \frac{4(x-2)(x+4)}{x-2} = 4(x+4)$$

$$8. \frac{x+2}{x-5} + \frac{6}{7} = \frac{7(x-4)}{x-5}$$

$$9. \frac{x-1}{x^2-1} + \frac{2}{5x+5} = \frac{7}{5(x+1)}$$

$$\frac{x-1}{(x-1)(x+1)} + \frac{2}{5(x+1)} = \frac{7}{5(x+1)}$$

$$= \frac{5+2}{5(x+1)}$$

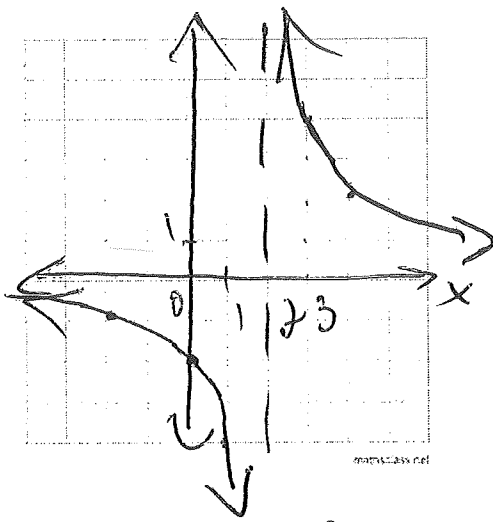
10. $\frac{7}{y} - \frac{2}{3y} = \frac{19}{3y}$

11. $\frac{7}{y-2} - \frac{11}{2-y} = \frac{18}{y-2}$
 $= \frac{7}{y-2} + \frac{11}{y-2}$

12. $\frac{3}{4b} - \frac{2}{5b} - \frac{1}{2b} = \frac{-3}{20b}$

13. **BIOLOGY** After a person eats something, the pH or acid level A of their mouth can be determined by the formula $A = \frac{-20.4t}{t^2 + 36} + 6.5$, where t is the number of minutes that have elapsed since the food was eaten. What would the acid level be after 30 minutes? ≈ 5.8

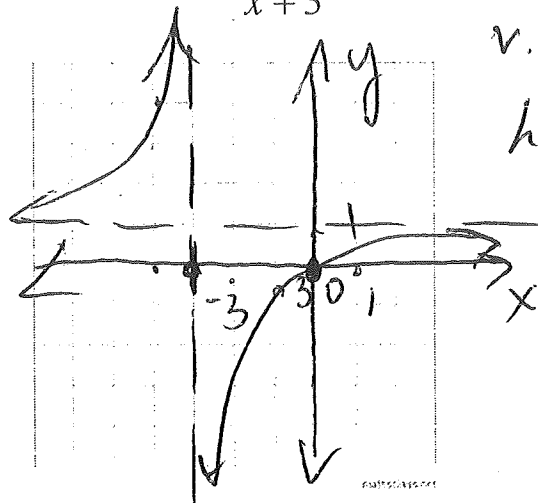
14. $f(x) = \frac{4}{x-2}$



v. as. $x=2$

h. as. $y=0$

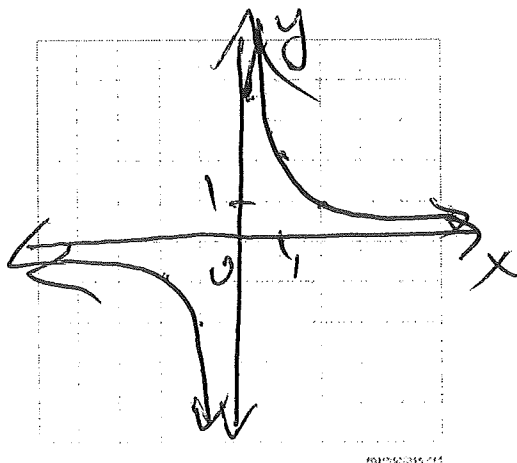
15. $f(x) = \frac{x}{x+3}$



v. as. $x=-3$

h. as. $y=1$

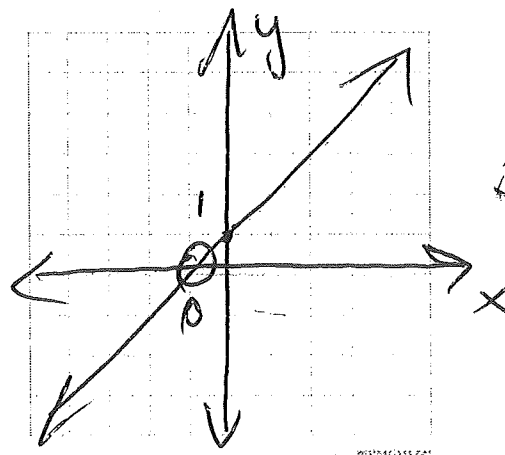
16. $f(x) = \frac{2}{x}$



v. as. $x=0$

h. as. $y=0$

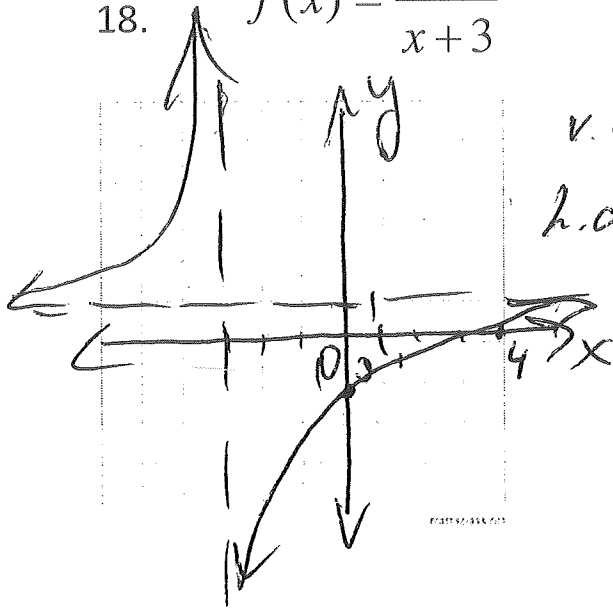
17. $f(x) = \frac{x^2 + 2x + 1}{x+1}$, $f(x) = \frac{(x+1)^2}{x+1}$



$f(x) = x+1$

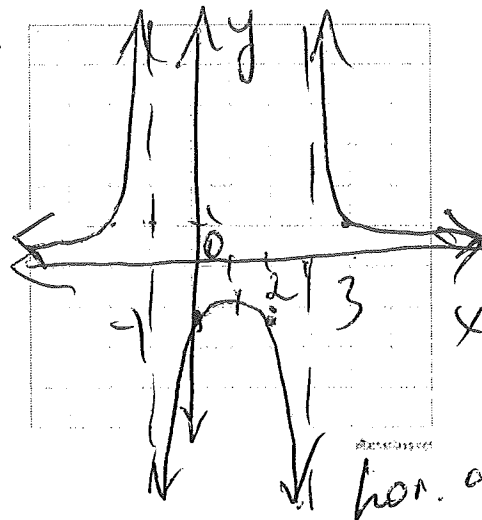
$x=-1$ is a hole.

18. $f(x) = \frac{x-4}{x+3}$



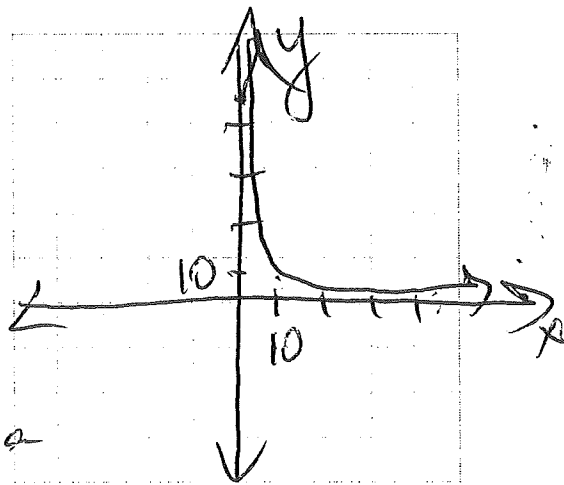
v. as.: $x = -3$
h. as.: $y = 1$

19. $f(x) = \frac{5}{(x+1)(x-3)}$



v. as.:
 $x = -1$
 $x = 3$
h. as.: $y = 0$

20. **SADWICHES** A group makes 45 sandwiches to take on a picnic. The number of sandwiches a person can eat depends on how many people go on the trip. Write and graph a function to illustrate this solution.



$x = \#$ of people on a picnic

$y = \#$ of sandwiches one person can eat

$$xy = 45, \quad \underline{x \geq 0}, \quad \underline{y \geq 0}$$

$$y = \frac{45}{x}$$

21. If y varies directly as x and $y = 21$ when $x = 7$, find x when $y = -5$. $\left(\frac{-5}{3}\right)$

22. If y varies inversely as x and $y = 9$ when $x = 2.5$, find y when $x = -0.6$. (-37.5)

23. If y varies inversely as x and $y = -4$ when $x = 8$, find y when $x = -121$. $\left(\frac{32}{121}\right)$ $\frac{32}{121}$

24. If y varies jointly as x and z and $x = 2$ and $z = 4$ when $y = 16$, find y when $x = 5$ and $z = 8$. (80)

$$\begin{aligned} y &= k \cdot x \cdot z \\ 16 &= k \cdot 2 \cdot 4 \\ k &= 2 \end{aligned}$$

$$\begin{aligned} y &= 2 \cdot x \cdot z \\ y &= 2 \cdot 5 \cdot 8 \end{aligned}$$

25. If y varies jointly as x and z and $y = 14$ when $x = 10$ and $z = 7$, find y when $x = 11$ and $z = 8$. (17.6)

26. **EMPLOYMENT** Chris's pay varies directly with how many lawns he mows. If his pay is \$65 for 5 yards, find his pay after he has mowed 13 yards.

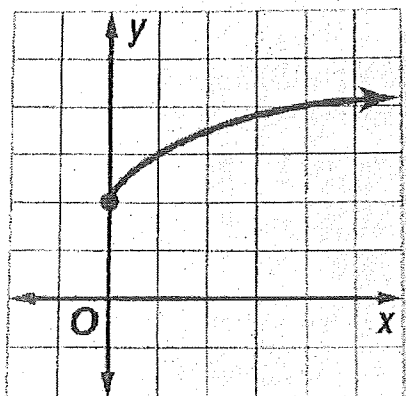
$$y = kx$$
$$65 = k \cdot 5; \quad k = 13$$

$$y = 13x$$

$$y = 13 \cdot 13 = \text{\$}169$$

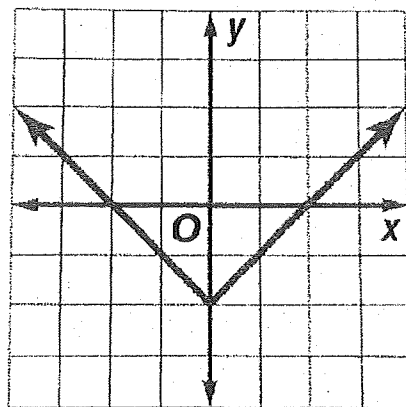
Identify the type of function.

27.



sq. root

28.



abs. v

Solve each equation or inequality. Check your solutions.

29. $\frac{3}{y} + \frac{7}{y} = 9$ $\left(\frac{10}{9}\right)$

30. $\frac{3x+2}{4} = \frac{9}{4} - \frac{(3-2x)}{6}$ $\left(3\right)$

$$9x + 6 = 27 - 2(3 - 2x)$$

$$9x + 6 = 27 - 6 + 4x$$

$$5x = 15$$

$$x = 3$$

31. $\frac{1}{r^2 - 1} = \frac{2}{r^2 + r - 2}$ $\left(0\right)$

32. $\frac{x}{x^2-1} + \frac{2}{x+1} = 1 + \frac{1}{2x-2}$

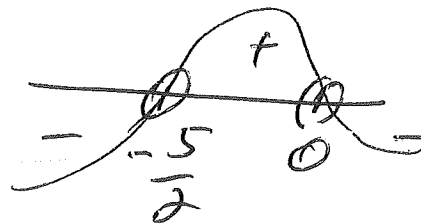
$\frac{3}{2}$

33. $\frac{1}{3b} - \frac{3}{4b} > \frac{1}{6}$

$\frac{4-9-2b}{12b} > 0$

$\frac{-5-2b}{12b} > 0$

$\frac{-(5+2b)}{12b} > 0$



$(-\frac{5}{2}, 0)$

34. **PUZZLES** Danielle can put a puzzle together in three hours. Aidan can put the same puzzle together in five hours. How long will it take them if they work together?

rate

D	A
$\frac{1}{3}$	$\frac{1}{5}$

$$\frac{1}{3}t + \frac{1}{5}t = 1$$

$$\frac{5t + 3t}{15} = 1$$

$$8t = 15$$

$$t = \frac{15}{8} \text{ hours}$$

1.875 hrs

Simplify.

35.
$$\frac{x^2 - 2x + 1}{y - 5} \div \frac{x - 1}{y^2 - 25} = (x - 1)(y + 5)$$

36.
$$\frac{\frac{x^2 - 1}{x^2 - 3x - 10}}{\frac{x^2 + 3x + 2}{x^2 - 12x + 35}} = \frac{(x - 7)(x - 1)}{(x + 2)^2}$$

$$\frac{(x-1)(x+1)(x-5)(x-7)}{(x-5)(x+2)(x+2)(x+1)}$$

$$37. \quad \frac{x-2}{x-1} + \frac{6}{7x-7} = \frac{7x-8}{7(x-1)}$$

$$38. \quad \frac{x}{x^2-9} + \frac{1}{2x+6} = \frac{3(x-1)}{2(x-3)(x+3)}$$

Solve.

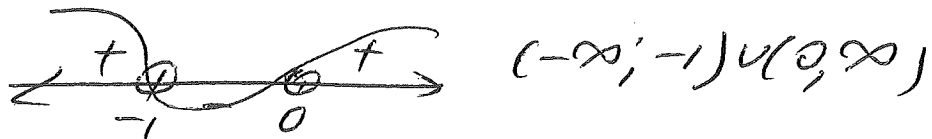
$$39. \quad \frac{2}{x-1} = 4 - \frac{x}{x-1} \quad (2)$$

40. $\frac{9}{28} + \frac{3}{z+2} = \frac{3}{4}$ (5)

41. $\frac{5}{7} + \frac{3}{t} > -\frac{2}{t}$
 $\frac{5}{7} + \frac{3}{t} + \frac{2}{t} > 0$

$$\frac{5t+5}{t} > 0$$

$$\frac{5(t+1)}{t} > 0$$



42. $x + \frac{12}{x} - 8 = 0$ (2; 6)

43. $\frac{5}{6} \frac{2m}{2m+3} = \frac{19}{6}$ $\left(\frac{-21}{20} \right)$

$$\begin{aligned} \underline{10m} + 15 - \underline{12m} &= \underline{38m} + 57 \\ -42 &= 40m \end{aligned}$$

44. $(2x)(2x+1) \left(\frac{x-3}{2x} = \frac{x-2}{2x+1} - \frac{1}{2} \right) \left(\frac{+\sqrt{6}}{2} \right)$

$$(2x+1)(x-3) = 2x(x-2) - (x)(2x+1)$$

$$= (2x^2 + x)$$

$$2x^2 - 6x + x - 3 = 2x^2 - 4x - 2x^2 - x$$

$$2x^2 - 5x - 3 = -5x$$

$$2x^2 - 3 = 0$$

$$2x^2 = 3$$

$$x^2 = 3/2$$

$$x = \pm \sqrt{3/2}$$

$$\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{6}}{2}$$